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Mark Scheme Summer 2009

GCE

GCE Mathematics (8371/8374; 9371/9374)

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Summer 2009

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June 2009
6663 Core Mathematics C1
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	$(3\sqrt{7})^2 = 63$	B1 (1)
(b)	$(8+\sqrt{5})(2-\sqrt{5}) = 16-5+2\sqrt{5}-8\sqrt{5}$	M1
	$= 11, \ -6\sqrt{5}$	A1, A1
		(3) **[4]**
(a)	B1 for 63 only	
(b)	M1 for an attempt to expand <u>their</u> brackets with $\geq$ 3 terms correct. They may collect the $\sqrt{5}$ terms to get $16-5-6\sqrt{5}$ Allow $-\sqrt{5}\times\sqrt{5}$ or $-\left(\sqrt{5}\right)^2$ or $-\sqrt{25}$ instead of the -5 These 4 values may appear in a list or table but they should have minus signs included **The next two marks should be awarded for the final answer but check that correct values follow from correct working. Do not use ISW rule** 1st A1 for 11 from 16 – 5 <u>or</u> $-6\sqrt{5}$ from $-8\sqrt{5}+2\sqrt{5}$ 2nd A1 for <u>both</u> 11 and $-6\sqrt{5}$. <u>S.C - Double sign error in expansion</u> For $16-5-2\sqrt{5}+8\sqrt{5}$ leading to 11 + … allow <u>one</u> mark	

Question Number	Scheme	Marks
Q2	$32 = 2^5$ or $2048 = 2^{11}$, $\sqrt{2} = 2^{1/2}$ or $\sqrt{2048} = (2048)^{\frac{1}{2}}$	B1, B1
	$a = \frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$	B1
		[3]
	1^{st} B1 for $32 = 2^5$ or $2048 = 2^{11}$ This should be explicitly seen: $32\sqrt{2} = 2^a$ followed by $2^5\sqrt{2} = 2^a$ is OK Even writing $32 \times 2 = 2^5 \times 2\left(= 2^6\right)$ is OK but simply writing $32 \times 2 = 2^6$ is NOT 2^{nd} B1 for $2^{\frac{1}{2}}$ or $(2048)^{\frac{1}{2}}$ seen. This mark may be implied 3^{rd} B1 for answer as written. **Need *a* =** … so $2^{\frac{11}{2}}$ is B0 $a = \frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$ with no working scores full marks. If $a = 5.5$ seen then award 3/3 unless it is clear that the value follows from totally incorrect work. Part solutions: e.g. $2^5\sqrt{2}$ scores the first B1. Special case: If $\sqrt{2} = 2^{1/2}$ is not explicitly seen, but the final answer includes $\frac{1}{2}$, e.g. $a = 2\frac{1}{2}$, $a = 4\frac{1}{2}$, the second B1 is given by implication.	

Question Number	Scheme	Marks
Q3 (a)	$\frac{dy}{dx} = 6x^2 - 6x^{-3}$	M1 A1 A1 (3)
(b)	$\frac{2x^4}{4} + \frac{3x^{-1}}{-1}(+C)$	M1 A1
	$\frac{x^4}{2} - 3x^{-1} + C$	A1 (3) **[6]**
(a)	M1 for an attempt to differentiate $x^n \to x^{n-1}$ 1st A1 for $6x^2$ 2nd A1 for $-6x^{-3}$ or $-\frac{6}{x^3}$ Condone $+ -6x^{-3}$ here. Inclusion of $+c$ scores A0 here.	
(b)	M1 for some attempt to integrate an x term of the given y. $x^n \to x^{n+1}$ 1st A1 for **both** x terms correct but unsimplified- as printed or better. Ignore $+c$ here 2nd A1 for both x terms correct and simplified and $+c$. Accept $-\frac{3}{x}$ but <u>NOT</u> $+ -3x^{-1}$ Condone the $+c$ appearing on the first (unsimplified) line but missing on the final (simplified) line Apply ISW if a correct answer is seen If part (b) is attempted first and this is clearly labelled then apply the scheme and allow the marks. Otherwise assume the first solution is for part (a).	

Question Number	Scheme	Marks
Q4 (a)	$5x > 10$, $x > 2$ [Condone $x > \frac{10}{2} = 2$ for M1A1]	M1, A1 (2)
(b)	$(2x+3)(x-4) = 0$, 'Critical values' are $-\frac{3}{2}$ and 4	M1, A1
	$-\frac{3}{2} < x < 4$	M1 A1ft (4)
(c)	$2 < x < 4$	B1ft (1) **[7]**
(a)	M1 for attempt to collect like terms on each side leading to $ax > b$, or $ax < b$, or $ax = b$ Must have a or b correct so eg $3x > 4$ scores M0	
(b)	1st M1 for an attempt to factorize or solve to find critical values. Method must potentially give 2 critical values 1st A1 for $-\frac{3}{2}$ and 4 seen. They may write $x < -\frac{3}{2}$, $x < 4$ and still get this A1 2nd M1 for choosing the "inside region" for their critical values 2nd A1ft follow through their 2 distinct critical values Allow $x > -\frac{3}{2}$ with "or" "," "$\cup$" " " $x < 4$ to score M1A0 but "and" or "$\cap$ " score M1A1 $x \in (-\frac{3}{2}, 4)$ is M1A1 but $x \in [-\frac{3}{2}, 4]$ is M1A0. Score M0A0 for a number line or graph only	
(c)	B1ft Allow if **a correct answer is seen** or follow through their answer to (a) and their answer to (b) but their answers to (a) and (b) must be regions. Do not follow through single values. If their follow through answer is the empty set accept $\varnothing$ or {} or equivalent in words If (a) or (b) are not given then score this mark for cao NB You may see $x<4$ (with anything or nothing in-between) $x < -1.5$ in (b) and empty set in (c) for B1ft **Do not award marks for part (b) if only seen in part (c)** Use of $\le$ instead of $<$ (or $\ge$ instead of $>$) loses one accuracy mark only, at first occurrence.	

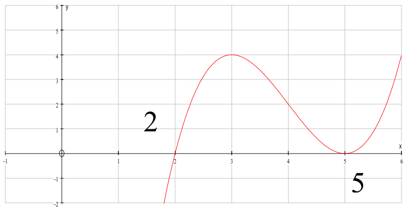
Question Number	Scheme	Marks
Q5 (a)	$a+9d=2400 \qquad a+39d=600$	M1
	$d=\frac{-1800}{30} \qquad d=-60 \qquad$ (accept $\pm$ 60 for A1)	M1 A1 (3)
(b)	$a-540=2400 \qquad a=2940$	M1 A1 (2)
(c)	Total $=\frac{1}{2}n\{2a+(n-1)d\}=\frac{1}{2}\times 40\times(5880+39\times -60)$ (ft values of a and d)	M1 A1ft
	$=\underline{70\ 800}$	A1cao (3) **[8]**
	Note: If the sequence is considered 'backwards', an equivalent solution may be given using d = 60 with a = 600 and l = 2940 for part (b). This can still score full marks. **Ignore labelling of (a) and (b)**	
(a)	1st M1 for an attempt to use 2400 and 600 in $a+(n-1)d$ formula. Must use both values i.e. need $a+pd=2400$ <u>and</u> $a+qd=600$ where p = 8 or 9 and q = 38 or 39 (any combination) 2nd M1 for an attempt to solve <u>their</u> 2 linear equations in a and d as far as d =… A1 for $d=\pm$ 60. Condone correct equations leading to d = 60 or $a+8d=2400$ and $a+38d=600$ leading to d = - 60. They should get penalised in (b) and (c). **NB This is a "one off" ruling for A1. Usually an A mark must follow from their work.** ALT 1st M1 for $(30d)=\pm(2400-600)$ 2nd M1 for $(d=)\pm\frac{(2400-600)}{30}$ A1 for $d=\pm$ 60 $a+9d=600$, $a+39d=2400$ **only** scores M0 BUT if they solve to find $d=\pm$ 60 then use ALT scheme above.	
(b)	M1 for use of <u>their</u> d in a **correct** linear equation to find a leading to a = … A1 their a must be compatible with their d so d = 60 must have a = 600 and d = -60, a = 2940 So for example they can have 2400 = a + 9(60) leading to a = … for M1 but it scores A0 Any approach using a list scores M1A1 for a correct a but M0A0 otherwise	
(c)	M1 for use of a correct S_n formula with n = 40 and at least one of a, d or l correct or correct ft. 1st A1ft for use of a correct S_{40} formula and both a, d or a, l correct or correct follow through ALT Total $=\frac{1}{2}n\{a+l\}=\frac{1}{2}\times 40\times(2940+600)$ (ft value of a) M1 A1ft 2nd A1 for 70800 only	

Question Number	Scheme	Marks
Q6	$b^2 - 4ac$ attempted, in terms of p.	M1
	$(3p)^2 - 4p = 0$ o.e.	A1
	Attempt to solve for p e.g. $p(9p-4)=0$ Must potentially lead to $p=k,\ k \neq 0$	M1
	$p = \frac{4}{9}$ (Ignore $p = 0$, if seen)	A1cso
		[4]
	1st M1 for an attempt to substitute into $b^2 - 4ac$ or $b^2 = 4ac$ with b or c correct Condone x's in one term only. This can be inside a square root as part of the quadratic formula for example. **Use of inequalities can score the M marks only** 1st A1 for any correct equation: $(3p)^2 - 4\times 1\times p = 0$ or better 2nd M1 for an attempt to factorize or solve their quadratic expression in p. Method must be sufficient to lead to their $p = \frac{4}{9}$. Accept factors or use of quadratic formula or $(p \pm \frac{2}{9})^2 = k^2$ (o.e. eg) $(3p \pm \frac{2}{3})^2 = k^2$ or equivalent work on <u>their</u> eqn. $9p^2 = 4p \Rightarrow \frac{9p^{\cancel{2}}}{\cancel{p}} = 4$ which would lead to $9p = 4$ is OK for this 2nd M1 ALT <u>Comparing coefficients</u> M1 for $(x+\alpha)^2 = x^2 + \alpha^2 + 2\alpha x$ and A1 for a correct equation eg $3p = 2\sqrt{p}$ M1 for forming solving leading to $\sqrt{p} = \frac{2}{3}$ or better <u>Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark</u> If the formula is quoted accept some correct substitution leading to a partially correct expression. If the formula is not quoted only award for a fully correct expression using their values.	

Question Number	Scheme	Marks
Q7 (a)	$(a_2 =)2k-7$	B1 (1)
(b)	$(a_3 =)2(2k-7)-7$ or $4k-14-7, = 4k-21$ (*)	M1, A1cso (2)
(c)	$(a_4 =)2(4k-21)-7 \quad (=8k-49)$	M1
	$\sum_{r=1}^{4} a_r = k + "(2k-7)" + (4k-21) + "(8k-49)"$	M1
	$k+(2k-7)+(4k-21)+(8k-49) = 15k-77 = 43 \qquad k=8$	M1 A1 (4) **[7]**
(b)	M1 must see 2(their a_2) - 7 or $2(2k-7)-7$ or $4k-14-7$. Their a_2 must be a function of k. A1cso must see the $2(2k-7)-7$ or $4k-14-7$ expression and the $4k$ - 21 with no incorrect working	
(c)	1st M1 for an attempt to find a_4 using the given rule. Can be awarded for $8k$ - 49 seen. **Use of formulae for the sum of an arithmetic series scores M0M0A0 for the next 3 marks.** 2nd M1 for attempting the sum of the 1st 4 terms. Must have "+" not just , or clear attempt to sum. Follow through their a_2 and a_4 provided they are linear functions of k. Must lead to linear expression in k. Condone use of their linear $a_3 \neq 4k-21$ here too. 3rd M1 for forming a linear equation in k using their sum and the 43 and attempt to solve for k as far as $pk=q$ A1 for $k = 8$ only so $k = \frac{120}{15}$ is A0 <u>Answer Only</u> (e.g. trial improvement) Accept $k = 8$ <u>only if</u> 8 + 9 + 11 + 15 = 43 is seen as well <u>Sum $a_2+a_3+a_4+a_5$</u> or <u>$a_2+a_3+a_4$</u> Allow: M1 if $8k$ - 49 is seen, M0 for the sum (since they are not adding the 1st 4 terms) then M1 if they use their sum along with the 43 to form a linear equation and attempt to solve but A0	

Question Number	Scheme	Marks
Q8 (a)	AB: $m = \frac{2-7}{8-6}$, $\left(= -\frac{5}{2}\right)$	B1
	Using $m_1 m_2 = -1$: $m_2 = \frac{2}{5}$	M1
	$y - 7 = \frac{2}{5}(x-6)$, $2x - 5y + 23 = 0$ (o.e. with integer coefficients)	M1, A1 (4)
(b)	Using $x = 0$ in the answer to (a), $y = \frac{23}{5}$ or 4.6	M1, A1ft (2)
(c)	Area of triangle = $\frac{1}{2} \times 8 \times \frac{23}{5} = \frac{92}{5}$ (o.e) e.g. $\left(18\frac{2}{5},\ 18.4,\ \frac{184}{10}\right)$	M1 A1 (2)
		[8]
(a)	B1 for an expression for the gradient of AB. Does not need the $= -2.5$ 1st M1 for use of the perpendicular gradient rule. Follow through their m 2nd M1 for the use of (6, 7) and their changed gradient to form an equation for l. Can be awarded for $\frac{y-7}{x-6} = \frac{2}{5}$ o.e. Alternative is to use (6, 7) in $y = mx + c$ to <u>find a value</u> for c. Score when $c = \ldots$ is reached. A1 for a correct equation in the required form and must have "= 0" and integer coefficients	
(b)	M1 for using $x = 0$ in their answer to part (a) e.g. $-5y + 23 = 0$ A1ft for $y = \frac{23}{5}$ provided that $x = 0$ clearly seen <u>or</u> C (0, 4.6). Follow through their equation in (a) If x =0, y = 4.6 are clearly seen but C is given as (4.6,0) apply ISW and award the mark. This A mark requires a simplified fraction or an exact decimal Accept their 4.6 marked on diagram next to C for M1A1ft	
(c)	M1 for $\frac{1}{2} \times 8 \times y_C$ so can follow through their y coordinate of C. A1 for 18.4 (o.e.) but their y coordinate of C must be positive <u>Use of 2 triangles or trapezium and triangle</u> Award M1 when an expression for area of OCB only is seen <u>Determinant approach</u> Award M1 when an expression containing $\frac{1}{2} \times 8 \times y_C$ is seen	

Question Number	Scheme	Marks
Q9 (a)	$\left[(3-4\sqrt{x})^2=\right]9-12\sqrt{x}-12\sqrt{x}+(-4)^2x$	M1
	$9x^{-\frac{1}{2}}+16x^{\frac{1}{2}}-24$	A1, A1 (3)
(b)	$\mathrm{f}'(x)=-\frac{9}{2}x^{-\frac{3}{2}},\ +\frac{16}{2}x^{-\frac{1}{2}}$	M1 A1, A1ft (3)
(c)	$\mathrm{f}'(9)=-\frac{9}{2}\times\frac{1}{27}+\frac{16}{2}\times\frac{1}{3}=-\frac{1}{6}+\frac{16}{6}=\frac{5}{2}$	M1 A1 (2) **[8]**
(a)	M1 for an attempt to expand $(3-4\sqrt{x})^2$ with at least 3 terms correct- as printed or better <u>Or</u> $9-k\sqrt{x}+16x$ $(k\neq 0)$. See also the MR rule below 1st A1 for their coefficient of $\sqrt{x}$ = 16. Condone writing $(\pm)9x^{(\pm)\frac{1}{2}}$ instead of $9x^{-\frac{1}{2}}$ 2nd A1 for B = - 24 or their constant term = - 24	
(b)	M1 for an attempt to differentiate an x term $x^n\to x^{n-1}$ 1st A1 for $-\frac{9}{2}x^{-\frac{3}{2}}$ <u>and</u> their constant B differentiated to zero. NB $-\frac{1}{2}\times 9x^{-\frac{3}{2}}$ is A0 2nd A1ft follow through their $Ax^{\frac{1}{2}}$ but can be scored without a value for A, i.e. for $\frac{A}{2}x^{-\frac{1}{2}}$	
(c)	M1 for some correct substitution of $x=9$ in <u>their</u> expression for $\mathrm{f}'(x)$ including an attempt at $(9)^{\pm\frac{k}{2}}$ (k odd) somewhere that leads to some appropriate multiples of $\frac{1}{3}$ or 3 A1 accept $\frac{15}{6}$ or any exact equivalent of 2.5 e.g. $\frac{45}{18},\frac{135}{54}$ or even $\frac{67.5}{27}$ <u>Misread (MR)</u> Only allow MR of the form $\frac{(3-k\sqrt{x})^2}{\sqrt{x}}$ N.B. Leads to answer in (c) of $\frac{k^2-1}{6}$ Score as M1A0A0, M1A1A1ft, M1A1ft	

Question Number	Scheme	Marks
Q10 (a)	$x(x^2-6x+9)$ $= x(x-3)(x-3)$	B1 M1 A1 (3)
(b)	 Shape  <u>Through</u> origin (<u>not</u> touching) Touching x-axis only once Touching at (3, 0), or 3 on x-axis [Must be on graph not in a table]	B1 B1 B1 B1ft (4)
(c)	 Moved horizontally (either way) (2, 0) and (5, 0), or 2 and 5 on x-axis	M1 A1 (2) **[9]**
(a) S.C.	B1 for correctly taking out a factor of x M1 for an attempt to factorize their 3TQ e.g. $(x+p)(x+q)$ where $\lvert pq\rvert = 9$. So $(x-3)(x+3)$ will score M1 but A0 A1 for a fully correct factorized expression - accept $x(x-3)^2$ If they "solve" use ISW If the only correct linear factor is $(x - 3)$, perhaps from factor theorem, award B0M1A0 Do not award marks for factorising in part (b) <u>**For the graphs**</u> "Sharp points" will lose the 1st B1 in (b) but otherwise be generous on shape Condone (0, 3) in (b) and (0, 2), (0,5) in (c) if the points are marked in the correct places.	
(b)	2nd B1 for a curve that starts or terminates at (0, 0) score B0 4th B1ft for a curve that touches (not crossing or terminating) at $(a, 0)$ where their $y = x(x-a)^2$	
(c)	M1 for their graph moved horizontally (only) <u>or</u> a fully correct graph Condone a partial stretch if ignoring their values looks like a simple translation A1 for their graph translated 2 to the right <u>and</u> crossing or touching the axis at 2 and 5 only **Allow a fully correct graph (as shown above) to score M1A1 whatever they have in (b)**	

Question Number	Scheme	Marks
Q11 (a)	$x = 2: \quad y = 8 - 8 - 2 + 9 = 7 \quad (*)$	B1 (1)
(b)	$\frac{dy}{dx} = 3x^2 - 4x - 1$	M1 A1
	$x = 2: \quad \frac{dy}{dx} = 12 - 8 - 1 (= 3)$	A1ft
	$y - 7 = 3(x - 2)$, $\quad \underline{y = 3x + 1}$	M1, A1 (5)
(c)	$m = -\frac{1}{3}$ $\quad$ (for $-\frac{1}{m}$ with their m)	B1ft
	$3x^2 - 4x - 1 = -\frac{1}{3}$, $\quad 9x^2 - 12x - 2 = 0$ or $x^2 - \frac{4}{3}x - \frac{2}{9} = 0$ (o.e.)	M1, A1
	$\left(x = \frac{12 + \sqrt{144 + 72}}{18}\right)$ $\left(\sqrt{216} = \sqrt{36}\sqrt{6} = 6\sqrt{6}\right)$ or $(3x - 2)^2 = 6 \rightarrow 3x = 2 \pm \sqrt{6}$	M1
	$x = \frac{1}{3}\left(2 + \sqrt{6}\right) \quad (*)$	A1cso (5)
		[11]
(a)	B1 there must be a clear attempt to substitute $x = 2$ leading to 7 e.g. $2^3 - 2 \times 2^2 - 2 + 9 = 7$	
(b)	1st M1 for an attempt to differentiate with at least one of the given terms fully correct. 1st A1 for a fully correct expression 2nd A1ft for sub. x= 2 in <u>their</u> $\frac{dy}{dx} (\neq y)$ accept for a correct expression e.g. $3 \times (2)^2 - 4 \times 2 - 1$ 2nd M1 for use of their "3" (provided it comes from their $\frac{dy}{dx} (\neq y)$ and x=2) to find equation of tangent. Alternative is to use (2, 7) in $y = mx + c$ to <u>find a value</u> for c. Award when $c = \ldots$ is seen. **No attempted use of $\frac{dy}{dx}$ in (b) scores 0/5**	
(c)	1st M1 for forming an equation from their $\frac{dy}{dx} (\neq y)$ and their $-\frac{1}{m}$ (must be changed from m) 1st A1 for a correct 3TQ all terms on LHS (condone missing =0) 2nd M1 for proceeding to $x =\ldots$ or $3x = \ldots$ by formula or completing the square for a 3TQ. Not factorising. Condone $\pm$ 2nd A1 for proceeding to given answer with no incorrect working seen. Can still have $\pm$.	
ALT	<u>Verify (for M1A1M1A1)</u> 1st M1 for attempting to square need $\geq$ 3 correct values in $\frac{4+6+4\sqrt{6}}{9}$, 1st A1 for $\frac{10+4\sqrt{6}}{9}$ 2nd M1 Dependent on 1st M1 in this case for substituting in all terms of their $\frac{dy}{dx}$ 2nd A1cso for cso <u>with a full comment</u> e.g. "the x co-ord of Q is ..."	

June 2009
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
Q1	$\int\left(2x+3x^{\frac{1}{2}}\right)\,dx=\frac{2x^2}{2}+\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$	M1 A1A1
	$\int_1^4\left(2x+3x^{\frac{1}{2}}\right)\,dx=\left[x^2+2x^{\frac{3}{2}}\right]_1^4=(16+2\times8)-(1+2)$	M1
	$=29$ (29 + C scores A0)	A1 (5)
		[5]
	1st M1 for attempt to integrate $x\to kx^2$ or $x^{\frac{1}{2}}\to kx^{\frac{3}{2}}$. 1st A1 for $\frac{2x^2}{2}$ or a simplified version. 2nd A1 for $\frac{3x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ or $\frac{3x\sqrt{x}}{\left(\frac{3}{2}\right)}$ or a simplified version. Ignore + C, if seen, but two correct terms and an extra non-constant term scores M1A1A0. 2nd M1 for correct use of correct limits ('top' – 'bottom'). Must be used in a 'changed function', not just the original. (The changed function may have been found by differentiation). Ignore 'poor notation' (e.g. missing integral signs) if the intention is clear. No working: The answer 29 with no working scores M0A0A0M1A0 (1 mark).	

Question Number	Scheme	Marks
Q2 (a)	$(7\times\ldots\times x)$ or $(21\times\ldots\times x^2)$ The 7 or 21 can be in 'unsimplified' form.	M1
	$(2+kx)^7 = 2^7 + 2^6\times 7\times kx + 2^5\times\binom{7}{2}k^2x^2$	
	$= 128;\quad +448kx,\quad +672k^2x^2$ [or $672(kx)^2$]	B1; A1, A1
	(If $672kx^2$ follows $672(kx)^2$, isw and allow A1)	(4)
(b)	$6\times 448k = 672k^2$	M1
	$k=4$ (Ignore $k=0$, if seen)	A1 (2) **[6]**
(a)	The terms can be 'listed' rather than added. Ignore any extra terms. M1 for <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or k) may be wrong or missing. <u>Allow</u> binomial coefficients such as $\binom{7}{1}, \left(\frac{7}{1}\right), \binom{7}{2}, {}^7C_1, {}^7C_2$. However, $448+kx$ or similar is M0. B1, A1, A1 for the <u>simplified</u> versions seen above. <u>Alternative</u>: Note that a factor 2^7 can be taken out first: $2^7\left(1+\frac{kx}{2}\right)^7$, but the mark scheme still applies. <u>Ignoring subsequent working (isw)</u>: Isw if necessary after correct working: e.g. $128+448kx+672k^2x^2$ M1 B1 A1 A1 $= 4+14kx+21k^2x^2$ isw (Full marks are still available in part (b)).	
(b)	M1 for equating their coefficient of x^2 to 6 times that of x… to get an equation in k, … <u>or</u> equating their coefficient of x to 6 times that of x^2, to get an equation in k. Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, e.g. $6\times 448k = 672k$, but beware $k=4$ following from this, which is A0. <u>An equation in k alone</u> is required for this M mark, so... e.g. $6\times 448kx = 672k^2x^2 \Rightarrow k=4$ or similar is M0 A0 (equation in coefficients only is never seen), but ... e.g. $6\times 448kx = 672k^2x^2 \Rightarrow 6\times 448k = 672k^2 \Rightarrow k=4$ will get M1 A1 (as coefficients rather than terms have now been considered). The mistake $2\left(1+\frac{kx}{2}\right)^7$ would give a maximum of 3 marks: M1B0A0A0, M1A1	

Question Number	Scheme	Marks
Q3 (a)	$\mathrm{f}(k) = -8$	B1 (1)
(b)	$\mathrm{f}(2) = 4 \Rightarrow \quad 4 = (6-2)(2-k) - 8$	M1
	So $\quad k = -1$	A1 (2)
(c)	$\mathrm{f}(x) = 3x^2 - (2+3k)x + (2k-8) \quad = 3x^2 + x - 10$	M1
	$= (3x - 5)(x+2)$	M1A1 (3)
		[6]
(b)	M1 for substituting $x = 2$ (<u>not</u> $x = -2$) and equating to 4 to form an equation in k. If the expression is expanded in this part, condone 'slips' for this M mark. Treat the omission of the -8 here as a 'slip' and allow the M mark. <u>Beware</u>: Substituting $x = -2$ and equating to 0 (M0 A0) also gives $k = -1$. <u>Alternative</u>; M1 for dividing by $(x-2)$, to get $3x +$ (function of k), with remainder as a function of k, and equating the remainder to 4. [Should be $3x + (4-3k)$, remainder $-4k$]. <u>No working</u>: $k = -1$ with no working scores M0 A0.	
(c)	1st M1 for multiplying out <u>and</u> substituting their (constant) value of k (in either order). The multiplying-out may occur earlier. Condone, for example, sign slips, but if the 4 (from part (b)) is included in the f(x) expression, this is M0. The 2nd M1 is still available. 2nd M1 for an attempt to factorise their three term quadratic (3TQ). A1 The correct answer, as a <u>product of factors</u>, is required. Allow $3\left(x - \frac{5}{3}\right)(x+2)$ Ignore following work (such as a solution to a quadratic equation). If the 'equation' is solved but factors are never seen, the 2nd M is not scored.	

Question Number	Scheme	Marks
Q4 (a)	$x = 2$ gives 2.236 (allow AWRT) Accept $\sqrt{5}$	B1
	$x = 2.5$ gives 2.580 (allow AWRT) Accept 2.58	B1 (2)
(b)	$\left(\frac{1}{2}\times\frac{1}{2}\right)$, $[(1.414+3)+2(1.554+1.732+1.957+2.236+2.580)]$	B1,[M1A1ft]
	$= 6.133$ (AWRT 6.13, even following minor slips)	A1 (4)
(c)	Overestimate	B1
	'Since the trapezia lie <u>above the curve</u>', or an equivalent explanation, or sketch of (one or more) trapezia above the curve on a diagram (or on the given diagram, in which case there should be reference to this). (Note that there must be some reference to a trapezium or trapezia in the explanation or diagram).	dB1 (2) **[8]**

(b)	B1 for $\frac{1}{2}\times\frac{1}{2}$ or equivalent. For the M mark, the first bracket must contain the 'first and last' values, and the second bracket (which must be multiplied by 2) must have no additional values. If the only mistake is to <u>omit</u> one of the values from the second bracket, this can be considered as a slip and the M mark can be allowed. Bracketing mistake: i.e. $\left(\frac{1}{2}\times\frac{1}{2}\right)(1.414+3)+2(1.554+1.732+1.957+2.236+2.580)$ scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). <u>Alternative</u>: Separate trapezia may be used, and this can be marked equivalently. $\left[\frac{1}{4}(1.414+1.554)+\frac{1}{4}(1.554+1.732)+\dots\dots\dots+\frac{1}{4}(2.580+3)\right]$ 1st A1ft for correct expression, ft their 2.236 and their 2.580
(c)	1st B1 for 'overestimate', ignoring earlier mistakes and ignoring any reasons given. 2nd B1 is dependent upon the 1st B1 (overestimate).

Question Number	Scheme	Marks
Q5 (a)	$324r^3 = 96$ or $r^3 = \frac{96}{324}$ or $r^3 = \frac{8}{27}$	M1
	$r = \frac{2}{3}$ (*)	A1cso (2)
(b)	$a\left(\frac{2}{3}\right)^2 = 324$ or $a\left(\frac{2}{3}\right)^5 = 96$ $a = \ldots,$ 729	M1, A1 (2)
(c)	$S_{15} = \frac{729\left(1-\left[\frac{2}{3}\right]^{15}\right)}{1-\frac{2}{3}}$, $= 2182.00\ldots$ (AWRT 2180)	M1A1ft, (3)
(d)	$S_\infty = \frac{729}{1-\frac{2}{3}}$, $= 2187$	M1, A1 (2)
		[9]

(a) M1 for forming an equation for r^3 based on 96 and 324 (e.g. $96r^3 = 324$ scores M1). The equation must involve multiplication/division rather than addition/subtraction.

A1 Do not penalise solutions with working in decimals, providing these are correctly rounded or truncated to at least 2dp and the final answer 2/3 is seen.

Alternative: (verification)

M1 Using $r^3 = \frac{8}{27}$ and multiplying 324 by this (or multiplying by $r = \frac{2}{3}$ three times).

A1 Obtaining 96 (cso). (A conclusion is not required).

$324 \times \left(\frac{2}{3}\right)^3 = 96$ (no real evidence of calculation) is not quite enough and scores M1 A0.

(b) M1 for the use of a correct formula or for 'working back' by dividing by $\frac{2}{3}$ (or by their r) twice from 324 (or 5 times from 96).

Exceptionally, allow M1 also for using $ar^3 = 324$ or $ar^6 = 96$ instead of $ar^2 = 324$ or $ar^5 = 96$, or for dividing by r three times from 324 (or 6 times from 96)… but no other exceptions are allowed.

(c) M1 for use of sum to 15 terms formula with values of a and r. If the wrong power is used, e.g. 14, the M mark is scored only if the correct sum formula is stated.

1st A1ft for a correct expression or correct ft their a with $r = \frac{2}{3}$.

2nd A1 for awrt 2180, even following 'minor inaccuracies'.

Condone missing brackets round the $\frac{2}{3}$ for the marks in part (c).

Alternative:

M1 for adding 15 terms and 1st A1ft for adding the 15 terms that ft from their a and $r = \frac{2}{3}$.

(d) M1 for use of correct sum to infinity formula with their a. For this mark, if a value of r different from the given value is being used, M1 can still be allowed providing $|r| < 1$.

Question Number	Scheme	Marks
Q6 (a)	$(x-3)^2-9+(y+2)^2-4=12$ Centre is (3, –2)	M1 A1, A1
	$(x-3)^2+(y+2)^2=12+"9"+"4"$ $r=\sqrt{12+"9"+"4"}=5$ (or $\sqrt{25}$)	M1 A1 (5)
(b)	$PQ=\sqrt{(7--1)^2+(-5-1)^2}$ or $\sqrt{8^2+6^2}$	M1
	= 10 = 2×radius, ∴diam. (N.B. For A1, need a comment or conclusion)	A1 (2)
	[ALT: midpt. of PQ $\left(\frac{7+(-1)}{2},\frac{1+(-5)}{2}\right)$: M1, = (3, –2) = centre: A1]	
	[ALT: eqn. of PQ $3x+4y-1=0$: M1, verify (3, –2) lies on this: A1]	
	[ALT: find two grads, e.g. PQ and P to centre: M1, equal ∴diameter: A1]	
	[ALT: show that point $S(-1,-5)$ or (7, 1) lies on circle: M1 because $\angle PSQ=90°$, semicircle∴diameter: A1]	
(c)	R must lie on the circle (angle in a semicircle theorem)… often implied by a diagram with R on the circle or by subsequent working)	B1
	$x=0\Rightarrow$ $y^2+4y-12=0$	M1
	$(y-2)(y+6)=0$ $y=.....$ (M is dependent on previous M)	dM1
	$y=-6$ or 2 (Ignore $y=-6$ if seen, and 'coordinates' are not required))	A1 (4) **[11]**
(a)	1st M1 for attempt to complete square. Allow $(x\pm3)^2\pm k$, or $(y\pm2)^2\pm k$, $k\neq0$. 1st A1 x-coordinate 3, 2nd A1 y-coordinate –2 2nd M1 for a full method leading to $r=$ …, with their 9 and their 4, 3rd A1 5 or $\sqrt{25}$ The 1st M can be implied by $(\pm3,\pm2)$ but a full method must be seen for the 2nd M. Where the 'diameter' in part (b) has clearly been used to answer part (a), no marks in (a), but in this case the M1 (not the A1) for part (b) can be given for work seen in (a). Alternative 1st M1 for comparing with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g,-f)$ directly. Condone sign errors for this M mark. 2nd M1 for using $r=\sqrt{g^2+f^2-c}$. Condone sign errors for this M mark.	
(c)	1st M1 for setting $x=0$ and getting a 3TQ in y by using eqn. of circle. 2nd M1 (dep.) for attempt to solve a 3TQ leading to at least one solution for y. Alternative 1: (Requires the B mark as in the main scheme) 1st M for using (3, 4, 5) triangle with vertices $(3,-2),(0,-2),(0,y)$ to get a linear or quadratic equation in y (e.g. $3^2+(y+2)^2=25$). 2nd M (dep.) as in main scheme, but may be scored by simply solving a linear equation. Alternative 2: (Not requiring realisation that R is on the circle) B1 for attempt at $m_{PR}\times m_{QR}=-1$, (NOT m_{PQ}) or for attempt at Pythag. in triangle PQR. 1st M1 for setting $x=0$, i.e. $(0, y)$, and proceeding to get a 3TQ in y. Then main scheme. Alternative 2 by 'verification': B1 for attempt at $m_{PR}\times m_{QR}=-1$, (NOT m_{PQ}) or for attempt at Pythag. in triangle PQR. 1st M1 for trying (0, 2). 2nd M1 (dep.) for performing all required calculations. A1 for fully correct working and conclusion.	

Question Number	Scheme	Marks
Q7 (i)	$\tan\theta = -1 \Rightarrow \quad \theta = -45, \quad 135$	B1, B1ft
	$\sin\theta = \frac{2}{5} \Rightarrow \quad \theta = 23.6, \quad 156.4$ (AWRT: 24, 156)	B1, B1ft (4)
(ii)	$4\sin x = \dfrac{3\sin x}{\cos x}$	M1
	$4\sin x\cos x = 3\sin x \quad \Rightarrow \quad \sin x(4\cos x - 3) = 0$ Other possibilities (after squaring): $\sin^2 x(16\sin^2 x - 7) = 0$, $(16\cos^2 x - 9)(\cos^2 x - 1) = 0$	M1
	$x = 0, \; 180$ seen	B1, B1
	$x = 41.4, \quad 318.6$ (AWRT: 41, 319)	B1, B1ft (6)
		[10]
(i)	1st B1 for -45 seen (α, where $\lvert\alpha\rvert < 90$) 2nd B1 for 135 seen, or ft ($180 + \alpha$) if α is negative, or ($\alpha - 180$) if α is positive. If $\tan\theta = k$ is obtained from wrong working, 2nd B1ft is still available. 3rd B1 for awrt 24 (β, where $\lvert\beta\rvert < 90$) 4th B1 for awrt 156, or ft ($180 - \beta$) if β is positive, or $-(180 + \beta)$ if β is negative. If $\sin\theta = k$ is obtained from wrong working, 4th B1ft is still available.	
(ii)	1st M1 for use of $\tan x = \dfrac{\sin x}{\cos x}$. Condone $\dfrac{3\sin x}{3\cos x}$. 2nd M1 for correct work leading to 2 factors (may be implied). 1st B1 for 0, 2nd B1 for 180. 3rd B1 for awrt 41 (γ, where $\lvert\gamma\rvert < 180$) 4th B1 for awrt 319, or ft ($360 - \gamma$). If $\cos\theta = k$ is obtained from wrong working, 4th B1ft is still available. N.B. Losing $\sin x = 0$ usually gives a maximum of 3 marks M1M0B0B0B1B1 Alternative: (squaring both sides) 1st M1 for squaring both sides and using a 'quadratic' identity. e.g. $16\sin^2\theta = 9(\sec^2\theta - 1)$ 2nd M1 for reaching a factorised form. e.g. $(16\cos^2\theta - 9)(\cos^2\theta - 1) = 0$ Then marks are equivalent to the main scheme. Extra solutions, if not rejected, are penalised as in the main scheme. For both parts of the question: Extra solutions outside required range: Ignore Extra solutions inside required range: For each pair of B marks, the 2nd B mark is lost if there are two correct values and one or more extra solution(s), e.g. $\tan\theta = -1 \Rightarrow \theta = 45, -45, 135$ is B1 B0 Answers in radians: Loses a maximum of 2 B marks in the whole question (to be deducted at the first and second occurrence).	

Question Number	Scheme	Marks
Q8 (a)	$\log_2 y = -3 \quad \Rightarrow \quad y = 2^{-3}$	M1
	$y = \frac{1}{8}$ or 0.125	A1 (2)
(b)	$32 = 2^5$ or $16 = 2^4$ or $512 = 2^9$	M1
	[or $\log_2 32 = 5\log_2 2$ or $\log_2 16 = 4\log_2 2$ or $\log_2 512 = 9\log_2 2$]	
	[or $\log_2 32 = \dfrac{\log_{10} 32}{\log_{10} 2}$ or $\log_2 16 = \dfrac{\log_{10} 16}{\log_{10} 2}$ or $\log_2 512 = \dfrac{\log_{10} 512}{\log_{10} 2}$]	
	$\log_2 32 + \log_2 16 = 9$	A1
	$(\log x)^2 = \ldots$ or $(\log x)(\log x) = \ldots$ (May not be seen explicitly, so M1 may be implied by later work, and the base may be 10 rather than 2)	M1
	$\log_2 x = 3 \quad \Rightarrow \quad x = 2^3 = 8$	A1
	$\log_2 x = -3 \quad \Rightarrow \quad x = 2^{-3} = \dfrac{1}{8}$	A1ft (5) **[7]**

(a)

M1 for <u>getting out of logs</u> correctly.
If done by change of base, $\log_{10} y = -0.903...$ is insufficient for the M1, but $y = 10^{-0.903}$ scores M1.

A1 for the <u>exact</u> answer, e.g. $\log_{10} y = -0.903 \Rightarrow y = 0.12502..$ scores M1 (implied) A0.

<u>Correct answer</u> with no working scores both marks.

<u>Allow</u> both marks for implicit statements such as $\log_2 0.125 = -3$.

(b)

1st M1 for expressing 32 or 16 or 512 as a power of 2, or for a change of base enabling evaluation of $\log_2 32$, $\log_2 16$ or $\log_2 512$ by calculator. (Can be implied by 5, 4 or 9 respectively).

1st A1 for 9 (exact).

2nd M1 for getting $(\log_2 x)^2$ = constant. The constant can be a log or a sum of logs.
If written as $\log_2 x^2$ instead of $(\log_2 x)^2$, allow the M mark <u>only</u> if subsequent work implies correct interpretation.

2nd A1 for 8 (exact). Change of base methods leading to a non-exact answer score A0.

3rd A1ft for an answer of $\dfrac{1}{\text{their } 8}$. An ft answer may be non-exact.

<u>Possible mistakes</u>:

$\log_2(2^9) = \log_2(x^2) \Rightarrow x^2 = 2^9 \Rightarrow x = \ldots$ scores M1A1(implied by 9)M0A0A0

$\log_2 512 = \log_2 x \times \log_2 x \Rightarrow x^2 = 512 \Rightarrow x = \ldots$ scores M0A0(9 never seen)M1A0A0

$\log_2 48 = (\log_2 x)^2 \Rightarrow (\log_2 x)^2 = 5.585 \Rightarrow x = 5.145,\ x = 0.194$ scores M0A0M1A0A1ft

<u>No working</u> (or 'trial and improvement'):
$x = 8$ scores M0 A0 M1 A1 A0

Question Number	Scheme	Marks
Q9 (a)	(Arc length =) $r\theta = r \times 1 = r$. Can be awarded by implication from later work, e.g. $3rh$ or $(2rh + rh)$ in the S formula. (Requires use of $\theta = 1$).	B1
	(Sector area =) $\frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times 1 = \frac{r^2}{2}$. Can be awarded by implication from later work, e.g. the correct volume formula. (Requires use of $\theta = 1$).	B1
	Surface area = 2 sectors + 2 rectangles + curved face $(= r^2 + 3rh)$ (See notes below for what is allowed here)	M1
	Volume = 300 = $\frac{1}{2}r^2h$	B1
	Sub for h: $S = r^2 + 3 \times \frac{600}{r} = r^2 + \frac{1800}{r}$ (*)	A1cso (5)
(b)	$\frac{dS}{dr} = 2r - \frac{1800}{r^2}$ or $2r - 1800r^{-2}$ or $2r + -1800r^{-2}$	M1A1
	$\frac{dS}{dr} = 0 \Rightarrow r^3 = \ldots$, $r = \sqrt[3]{900}$, or AWRT 9.7 (NOT -9.7 or ± 9.7)	M1, A1 (4)
(c)	$\frac{d^2S}{dr^2} = \ldots.$ and consider sign, $\frac{d^2S}{dr^2} = 2 + \frac{3600}{r^3} > 0$ so point is a minimum	M1, A1ft (2)
(d)	$S_{min} = (9.65\ldots)^2 + \frac{1800}{9.65\ldots}$	
	(Using their value of r, however found, in the <u>given</u> S formula)	M1
	$= 279.65\ldots$ (AWRT: 280) (Dependent on full marks in part (b))	A1 (2) **[13]**
(a)	M1 for attempting a formula (with terms added) for surface area. May be incomplete or wrong and may have extra term(s), but must have an r^2 (or $r^2\theta$) term and an rh (or $rh\theta$) term.	
(b)	<u>In parts (b), (c) and (d), ignore labelling of parts</u> 1st M1 for attempt at differentiation (one term is sufficient) $r^n \to kr^{n-1}$ 2nd M1 for setting their derivative (a 'changed function') = 0 and solving as far as $r^3 = \ldots$ (depending upon their 'changed function', this could be $r = \ldots$ or $r^2 = \ldots$, etc., but the algebra <u>must deal with a negative power</u> of r and should be sound apart from possible <u>sign</u> errors, so that $r^n = \ldots$ is consistent with their derivative).	
(c)	M1 for attempting second derivative (one term is sufficient) $r^n \to kr^{n-1}$, <u>and considering its sign</u>. Substitution of a value of r is not required. (<u>Equating it to zero is M0</u>). A1ft for a correct second derivative (or correct ft from their first derivative) <u>and</u> a valid reason (e.g. > 0), <u>and</u> conclusion. The actual <u>value</u> of the second derivative, if found, can be ignored. To score this mark as ft, their second derivative must indicate a minimum. <u>Alternative</u>: M1: Find <u>value</u> of $\frac{dS}{dr}$ on each side of their value of r and consider sign. A1ft: Indicate sign change of negative to positive for $\frac{dS}{dr}$, and conclude minimum. <u>Alternative</u>: M1: Find <u>value</u> of S on each side of their value of r and compare with their 279.65. A1ft: Indicate that both values are more than 279.65, and conclude minimum.	

June 2009
6665 Core Mathematics C3
Mark Scheme

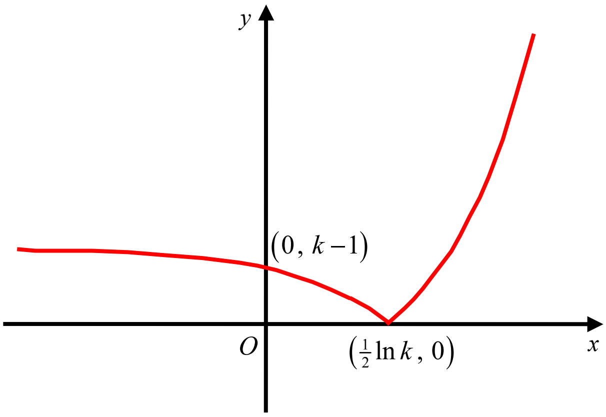
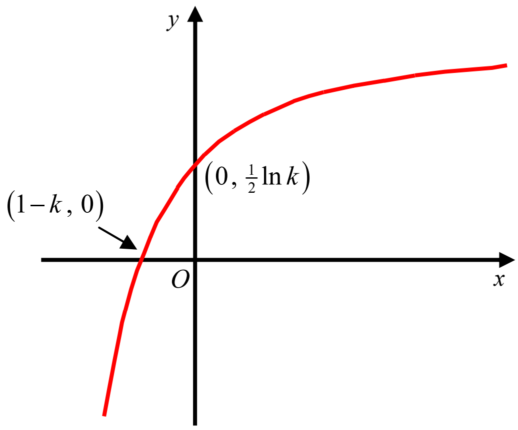
Question Number	Scheme		Marks
Q1 (a)	Iterative formula: $x_{n+1} = \dfrac{2}{(x_n)^2} + 2$, $x_0 = 2.5$		
	$x_1 = \dfrac{2}{(2.5)^2} + 2$	An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320	M1
	$x_1 = 2.32$ $x_2 = 2.371581451...$	Both $x_1 = 2.32(0)$ and $x_2 =$ awrt 2.372	A1
	$x_3 = 2.355593575...$ $x_4 = 2.360436923...$	Both $x_3 =$ awrt 2.356 and $x_4 =$ awrt 2.360 or 2.36	A1 cso
			(3)
(b)	Let $\mathrm{f}(x) = -x^3 + 2x^2 + 2 = 0$		
	$\mathrm{f}(2.3585) = 0.00583577...$	Choose suitable interval for x, e.g. [2.3585, 2.3595] or tighter	M1
	$\mathrm{f}(2.3595) = -0.00142286...$	any one value awrt 1 sf or truncated 1 sf	dM1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	both values correct, sign change and conclusion	A1
		At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states "change of sign, hence root".	(3)
			[6]

Question Number	Scheme		Marks
Q2 (a)	$\cos^2\theta + \sin^2\theta = 1 \quad (\div \cos^2\theta)$		
	$\underline{\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}}$	Dividing $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$ to give <u>underlined</u> equation.	M1
	$1 + \tan^2\theta = \sec^2\theta$		
	$\tan^2\theta = \sec^2\theta - 1$ (as required) **AG**	Complete proof. No errors seen.	A1 cso
			(2)
(b)	$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2, \quad (\text{eqn } *) \qquad 0 \le \theta < 360^\circ$		
	$2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$	Substituting $\tan^2\theta = \sec^2\theta - 1$ into eqn * to get a quadratic in $\sec\theta$ only	M1
	$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$		
	$\underline{3\sec^2\theta + 4\sec\theta - 4} = 0$	Forming a three term "one sided" quadratic expression in $\sec\theta$.	M1
	$(\sec\theta + 2)(3\sec\theta - 2) = 0$	Attempt to factorise or solve a quadratic.	M1
	$\sec\theta = -2$ or $\sec\theta = \frac{2}{3}$		
	$\dfrac{1}{\cos\theta} = -2$ or $\dfrac{1}{\cos\theta} = \dfrac{2}{3}$		
	$\underline{\cos\theta = -\frac{1}{2}}$; or $\cos\theta = \frac{3}{2}$	$\underline{\cos\theta = -\frac{1}{2}}$	A1;
	$\alpha = 120^\circ$ or $\alpha =$ no solutions		
	$\theta_1 = \underline{120^\circ}$	$\underline{120^\circ}$	<u>A1</u>
	$\theta_2 = 240^\circ$	$\underline{240^\circ}$ or $\theta_2 = 360^\circ - \theta_1$ when solving using $\cos\theta = \ldots$	B1 $\sqrt{}$
	$\theta = \{120^\circ, 240^\circ\}$	Note the final A1 mark has been changed to a B1 mark.	(6)
			[8]

Question Number	Scheme		Marks
Q3	$P = 80\mathrm{e}^{\frac{t}{5}}$		
(a)	$t = 0 \Rightarrow P = 80\mathrm{e}^{\frac{0}{5}} = 80(1) = \underline{80}$	$\underline{80}$	B1
			(1)
(b)	$P = 1000 \Rightarrow 1000 = 80\mathrm{e}^{\frac{t}{5}} \Rightarrow \frac{1000}{80} = \mathrm{e}^{\frac{t}{5}}$	Substitutes $P = 1000$ and rearranges equation to make $\mathrm{e}^{\frac{t}{5}}$ the subject.	M1
	$\therefore\ t = 5\ln\left(\frac{1000}{80}\right)$		
	$t = 12.6286...$	awrt 12.6 or 13 years	A1
		Note $t = 12$ or $t =$ awrt $12.6 \Rightarrow t = 12$ will score A0	(2)
(c)	$\frac{\mathrm{d}P}{\mathrm{d}t} = 16\mathrm{e}^{\frac{t}{5}}$	$k\mathrm{e}^{\frac{1}{5}t}$ and $k \neq 80$.	M1
		$16\mathrm{e}^{\frac{1}{5}t}$	A1
			(2)
(d)	$50 = 16\mathrm{e}^{\frac{t}{5}}$		
	$\therefore\ t = 5\ln\left(\frac{50}{16}\right)$ $\{= 5.69717...\}$	Using $50 = \frac{dP}{dt}$ and an attempt to solve to find the value of t or $\frac{t}{5}$.	M1
	$P = 80\mathrm{e}^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$ or $P = 80\mathrm{e}^{\frac{1}{5}(5.69717...)}$	Substitutes their value of t back into the equation for P.	dM1
	$P = \frac{80(50)}{16} = \underline{250}$	$\underline{250}$ or awrt 250	A1
			(3)
			[8]

Question Number	Scheme		Marks
Q4 (i)(a)	$y = x^2 \cos 3x$		
	Apply product rule: $\left\{\begin{array}{ll} u = x^2 & v = \cos 3x \\ \dfrac{du}{dx} = 2x & \dfrac{dv}{dx} = -3\sin 3x \end{array}\right\}$		
	$\dfrac{dy}{dx} = 2x\cos 3x - 3x^2 \sin 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x \cos 3x \pm \beta x^2 \sin 3x$	M1
		Any one term correct	A1
		Both terms correct and no further simplification to terms in $\cos \alpha x^2$ or $\sin \beta x^3$.	A1
			(3)
(b)	$y = \dfrac{\ln(x^2+1)}{x^2+1}$		
	$u = \ln(x^2+1) \Rightarrow \dfrac{du}{dx} = \dfrac{2x}{x^2+1}$	$\ln(x^2+1) \to \dfrac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \to \dfrac{2x}{x^2+1}$	A1
	Apply quotient rule: $\left\{\begin{array}{ll} u = \ln(x^2+1) & v = x^2+1 \\ \dfrac{du}{dx} = \dfrac{2x}{x^2+1} & \dfrac{dv}{dx} = 2x \end{array}\right\}$		
	$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{2x}{x^2+1}\right)(x^2+1) - 2x\ln(x^2+1)}{(x^2+1)^2}$	Applying $\dfrac{vu' - uv'}{v^2}$	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)
	$\left\{\dfrac{dy}{dx} = \dfrac{2x - 2x\ln(x^2+1)}{(x^2+1)^2}\right\}$	{Ignore subsequent working.}	

Question Number	Scheme		Marks
(ii)	$y = \sqrt{4x+1},\ \ x > -\frac{1}{4}$		
	At P, $y = \sqrt{4(2)+1} = \underline{\sqrt{9}} = \underline{3}$	At P, $y = \underline{\sqrt{9}}$ or $\underline{3}$	B1
	$\frac{dy}{dx} = \underline{\frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)}$	$\pm k(4x+1)^{-\frac{1}{2}}$ $2(4x+1)^{-\frac{1}{2}}$	M1* A1 aef
	$\frac{dy}{dx} = \frac{2}{(4x+1)^{\frac{1}{2}}}$		
	At P, $\frac{dy}{dx} = \frac{2}{(4(2)+1)^{\frac{1}{2}}}$	Substituting $x = 2$ into an equation involving $\frac{dy}{dx}$;	M1
	Hence m(**T**) $= \frac{2}{3}$		
	Either **T**: $y - 3 = \frac{2}{3}(x-2)$; or $y = \frac{2}{3}x + c$ and $3 = \frac{2}{3}(2) + c \Rightarrow c = 3 - \frac{4}{3} = \frac{5}{3}$;	$y - y_1 = m(x-2)$ or $y - y_1 = m(x - \text{their stated } x)$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient', their x and their y_1.	dM1*;
	Either **T**: $3y - 9 = 2(x-2)$;		
	T: $3y - 9 = 2x - 4$		
	T: $\underline{2x - 3y + 5 = 0}$	$\underline{2x - 3y + 5 = 0}$ Tangent must be stated in the form $ax + by + c = 0$, where a, b and c are integers.	A1
			(6)
	or **T**: $y = \frac{2}{3}x + \frac{5}{3}$		
	T: $3y = 2x + 5$		
	T: $\underline{2x - 3y + 5 = 0}$		
			[13]

Question Number	Scheme		Marks
Q5 (a)		Curve retains shape when $x > \frac{1}{2}\ln k$	B1
		Curve reflects through the x-axis when $x < \frac{1}{2}\ln k$	B1
		$(0,\ k-1)$ and $(\frac{1}{2}\ln k,\ 0)$ marked in the correct positions.	B1
			(3)
(b)		Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)	B1
		$(1-k,\ 0)$ and $(0,\ \frac{1}{2}\ln k)$	B1
			(2)
(c)	Range of f: $\underline{\mathrm{f}(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$	Either $\underline{\mathrm{f}(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$ or $\underline{\mathrm{f} > -k}$ or $\underline{\text{Range} > -k}$.	B1
			(1)
(d)	$y = \mathrm{e}^{2x} - k \Rightarrow y + k = \mathrm{e}^{2x}$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \ln(y+k) = 2x$ $\Rightarrow \frac{1}{2}\ln(y+k) = x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $\mathrm{f}^{-1}(x) = \underline{\frac{1}{2}\ln(x+k)}$	$\underline{\frac{1}{2}\ln(x+k)}$ or $\underline{\ln\sqrt{(x+k)}}$	<u>A1</u> cao
			(3)
(e)	$\mathrm{f}^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$	Either $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$ or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer	B1 $\sqrt{\ }$
			(1)
			[10]

Question Number	Scheme		Marks
Q6 (a)	$A = B \Rightarrow \cos(A + A) = \cos 2A = \underline{\cos A\cos A - \sin A\sin A}$	Applies $A = B$ to $\cos(A+B)$ to give the underlined equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$	M1
	$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives		
	$\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required)	Complete proof, with a link between LHS and RHS. No errors seen.	A1 AG
			(2)
(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$	Eliminating y correctly.	M1
	$3\sin 2x = 4\left(\dfrac{1-\cos 2x}{2}\right) - 2\cos 2x$	Using result in part (a) to substitute for $\sin^2 x$ as $\dfrac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\dfrac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles.	M1
	$3\sin 2x = 2(1 - \cos 2x) - 2\cos 2x$		
	$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$		
	$3\sin 2x + 4\cos 2x = 2$	Rearranges to give correct result	A1 AG
			(3)
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$		
	$3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$		
	Equate $\sin 2x$: $3 = R\sin\alpha$ Equate $\cos 2x$: $4 = R\cos\alpha$		
	$R = \sqrt{3^2 + 4^2}; = \sqrt{25} = 5$	$R = 5$	B1
	$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...^\circ$	$\tan\alpha = \pm\frac{3}{4}$ or $\tan\alpha = \pm\frac{4}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{4}{\text{their } R}$	M1
		awrt 36.87	A1
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$		
			(3)

Question Number	Scheme		Marks
(d)	$3\sin 2x + 4\cos 2x = 2$		
	$5\cos(2x - 36.87) = 2$		
	$\cos(2x - 36.87) = \frac{2}{5}$	$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$	M1
	$(2x - 36.87) = 66.42182...^{\circ}$	awrt 66	A1
	$(2x - 36.87) = 360 - 66.42182...^{\circ}$		
	Hence, $x = 51.64591...^{\circ},\ 165.22409...^{\circ}$	One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3	A1
		Both awrt 51.6 AND awrt 165.2	A1
			(4)
		If there are any EXTRA solutions inside the range $0 \le x < 180^{\circ}$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \le x < 180^{\circ}$.	
			[12]

Question Number	Scheme		Marks
Q7	$\mathrm{f}(x) = 1 - \dfrac{2}{(x+4)} + \dfrac{x-8}{(x-2)(x+4)}$ $x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$		
(a)	$\mathrm{f}(x) = \dfrac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$	An attempt to combine to one fraction Correct result of combining all three fractions	M1 A1
	$= \dfrac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$		
	$= \dfrac{x^2 + x - 12}{[(x+4)(x-2)]}$	Simplifies to give the correct numerator. Ignore omission of denominator	A1
	$= \dfrac{(x+4)(x-3)}{[(x+4)(x-2)]}$	An attempt to factorise the numerator.	dM1
	$= \dfrac{(x-3)}{(x-2)}$	Correct result	A1 cso **AG** (5)
(b)	$\mathrm{g}(x) = \dfrac{\mathrm{e}^x - 3}{\mathrm{e}^x - 2} \quad x \in \mathbb{R},\ x \neq \ln 2.$		
	Apply quotient rule: $\left\{ \begin{array}{ll} u = \mathrm{e}^x - 3 & v = \mathrm{e}^x - 2 \\ \dfrac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x & \dfrac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^x \end{array} \right\}$		
	$\mathrm{g}'(x) = \dfrac{\mathrm{e}^x(\mathrm{e}^x - 2) - \mathrm{e}^x(\mathrm{e}^x - 3)}{(\mathrm{e}^x - 2)^2}$	Applying $\dfrac{vu' - uv'}{v^2}$ Correct differentiation	M1 A1
	$= \dfrac{\mathrm{e}^{2x} - 2\mathrm{e}^x - \mathrm{e}^{2x} + 3\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$		
	$= \dfrac{\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$	Correct result	A1 **AG** cso (3)

Question Number	Scheme		Marks
(c)	$g'(x) = 1 \Rightarrow \dfrac{e^x}{(e^x - 2)^2} = 1$		
	$e^x = (e^x - 2)^2$	Puts their differentiated numerator equal to their denominator.	M1
	$e^x = e^{2x} - 2e^x - 2e^x + 4$		
	$\underline{e^{2x} - 5e^x + 4} = 0$	$\underline{e^{2x} - 5e^x + 4}$	A1
	$(e^x - 4)(e^x - 1) = 0$	Attempt to factorise or solve quadratic in e^x	M1
	$e^x = 4$ or $e^x = 1$		
	$x = \ln 4$ or $x = 0$	both $x = 0,\ \ln 4$	A1
			(4)
			[12]

Question Number	Scheme		Marks
Q8 (a)	$\sin 2x = \underline{2\sin x\cos x}$	$\underline{2\sin x\cos x}$	B1 aef (1)
(b)	$\operatorname{cosec} x - 8\cos x = 0\,,\quad 0 < x < \pi$		
	$\dfrac{1}{\sin x} - 8\cos x = 0$	Using $\operatorname{cosec} x = \dfrac{1}{\sin x}$	M1
	$\dfrac{1}{\sin x} = 8\cos x$		
	$1 = 8\sin x\cos x$		
	$1 = 4(2\sin x\cos x)$		
	$1 = 4\sin 2x$		
	$\underline{\sin 2x = \tfrac{1}{4}}$	$\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$	M1
		$\underline{\sin 2x = \tfrac{1}{4}}$	$\underline{\text{A1}}$
	Radians $2x = \{0.25268..., 2.88891...\}$ Degrees $2x = \{14.4775..., 165.5225...\}$		
	Radians $x = \{0.12634..., 1.44445...\}$ Degrees $x = \{7.23875..., 82.76124...\}$	Either arwt 7.24 or 82.76 or 0.13 or 1.44 or 1.45 or awrt 0.04π or awrt 0.46π.	A1
		Both $\underline{0.13}$ and $\underline{1.44}$	A1 cao (5)
		Solutions for the final two A marks must be given in x only. If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 < x < \pi$.	**[6]**

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6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
Q1	$\mathrm{f}(x)=\dfrac{1}{\sqrt{(4+x)}}=(4+x)^{-\frac{1}{2}}$	M1
	$=(4)^{-\frac{1}{2}}(1+\ \dots\ )^{\cdots}$ $\qquad \dfrac{1}{2}(1+\ \dots\ )^{\cdots}$ or $\dfrac{1}{2\sqrt{(1+\dots)}}$	B1
	$=\ \dots\left(1+(-\tfrac{1}{2})\left(\dfrac{x}{4}\right)+\dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2}\left(\dfrac{x}{4}\right)^2+\dfrac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}\left(\dfrac{x}{4}\right)^3+\ \dots\right)$	M1 A1ft
	ft their $\left(\dfrac{x}{4}\right)$	
	$=\dfrac{1}{2}-\dfrac{1}{16}x,+\dfrac{3}{256}x^2-\dfrac{5}{2048}x^3+\ \dots$	A1, A1 (6)
		[6]
	Alternative	
	$\mathrm{f}(x)=\dfrac{1}{\sqrt{(4+x)}}=(4+x)^{-\frac{1}{2}}$	M1
	$=\underline{4^{-\frac{1}{2}}}+(-\tfrac{1}{2})4^{-\frac{3}{2}}x+\dfrac{(-\frac{1}{2})(-\frac{3}{2})}{1.2}4^{-\frac{5}{2}}x^2+\dfrac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{1.2.3}4^{-\frac{7}{2}}x^3+\ \dots$	<u>B1</u> M1 A1
	$=\dfrac{1}{2}-\dfrac{1}{16}x,+\dfrac{3}{256}x^2-\dfrac{5}{2048}x^3+\ \dots$	A1, A1 (6)

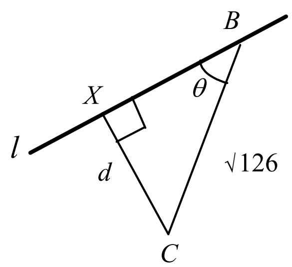
Question Number	Scheme		Marks	
Q2 (a)	1.14805	awrt 1.14805	B1	(1)
(b)	$A \approx \frac{1}{2}\times\frac{3\pi}{8}(\;\ldots\;)$		B1	
	$= \;\ldots\;\left(3+2\left(2.77164+2.12132+1.14805\right)+0\right)$	0 can be implied	M1	
	$= \frac{3\pi}{16}\left(3+2\left(2.77164+2.12132+1.14805\right)\right)$	ft their (a)	A1ft	
	$= \frac{3\pi}{16}\times 15.08202\;\ldots\; = 8.884$	cao	A1	(4)
(c)	$\int 3\cos\left(\frac{x}{3}\right)\mathrm{d}x = \frac{3\sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$		M1 A1	
	$= 9\sin\left(\frac{x}{3}\right)$			
	$A = \left[9\sin\left(\frac{x}{3}\right)\right]_0^{\frac{3\pi}{2}} = 9-0=9$	cao	A1	(3)
				[8]

Question Number	Scheme	Marks
Q3 (a)	$\mathrm{f}(x)=\dfrac{4-2x}{(2x+1)(x+1)(x+3)}=\dfrac{A}{2x+1}+\dfrac{B}{x+1}+\dfrac{C}{x+3}$	
	$4-2x=A(x+1)(x+3)+B(2x+1)(x+3)+C(2x+1)(x+1)$	M1
	A method for evaluating one constant	M1
	$x\to-\frac{1}{2}$, $5=A\left(\frac{1}{2}\right)\left(\frac{5}{2}\right)\Rightarrow A=4$ any one correct constant	A1
	$x\to-1$, $6=B(-1)(2)\Rightarrow B=-3$	
	$x\to-3$, $10=C(-5)(-2)\Rightarrow C=1$ all three constants correct	A1 (4)
(b)	(i) $\displaystyle\int\left(\frac{4}{2x+1}-\frac{3}{x+1}+\frac{1}{x+3}\right)\mathrm{d}x$	
	$=\dfrac{4}{2}\ln(2x+1)-3\ln(x+1)+\ln(x+3)+C$ A1 two ln terms correct	M1 A1ft
	All three ln terms correct and "$+C$" ; ft constants	A1ft (3)
	(ii) $\left[2\ln(2x+1)-3\ln(x+1)+\ln(x+3)\right]_0^2$	
	$=(2\ln 5-3\ln 3+\ln 5)-(2\ln 1-3\ln 1+\ln 3)$	M1
	$=3\ln 5-4\ln 3$	
	$=\ln\left(\dfrac{5^3}{3^4}\right)$	M1
	$=\ln\left(\dfrac{125}{81}\right)$	A1 (3)
		[10]

Question Number	Scheme		Marks	
Q4 (a)	$\mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}-2y\,\mathrm{e}^{-2x}=2+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$	A1 correct RHS	M1 A1	
	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(y\,\mathrm{e}^{-2x}\right)=\mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}-2y\,\mathrm{e}^{-2x}$		B1	
	$\left(\mathrm{e}^{-2x}-2y\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}=2+2y\,\mathrm{e}^{-2x}$		M1	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2+2y\,\mathrm{e}^{-2x}}{\mathrm{e}^{-2x}-2y}$		A1	(5)
(b)	At P , $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2+2\,\mathrm{e}^{0}}{\mathrm{e}^{0}-2}=-4$		M1	
	Using $mm'=-1$			
	$m'=\dfrac{1}{4}$		M1	
	$y-1=\dfrac{1}{4}(x-0)$		M1	
	$x-4y+4=0$	or any integer multiple	A1	(4)
				[9]
	Alternative for (a) *differentiating implicitly with respect to y*.			
	$\mathrm{e}^{-2x}-2y\,\mathrm{e}^{-2x}\dfrac{\mathrm{d}x}{\mathrm{d}y}=2\dfrac{\mathrm{d}x}{\mathrm{d}y}+2y$	A1 correct RHS	M1 A1	
	$\dfrac{\mathrm{d}}{\mathrm{d}y}\left(y\,\mathrm{e}^{-2x}\right)=\mathrm{e}^{-2x}-2y\,\mathrm{e}^{-2x}\dfrac{\mathrm{d}x}{\mathrm{d}y}$		B1	
	$\left(2+2y\,\mathrm{e}^{-2x}\right)\dfrac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{e}^{-2x}-2y$		M1	
	$\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{\mathrm{e}^{-2x}-2y}{2+2y\,\mathrm{e}^{-2x}}$			
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2+2y\,\mathrm{e}^{-2x}}{\mathrm{e}^{-2x}-2y}$		A1	(5)

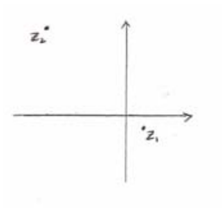
Question Number	Scheme	Marks
Q5 (a)	$\dfrac{dx}{dt} = -4\sin 2t$, $\dfrac{dy}{dt} = 6\cos t$	B1, B1
	$\dfrac{dy}{dx} = -\dfrac{6\cos t}{4\sin 2t}$ $\left(= -\dfrac{3}{4\sin t} \right)$	M1
	At $t = \dfrac{\pi}{3}$, $m = -\dfrac{3}{4 \times \frac{\sqrt{3}}{2}} = -\dfrac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87	A1 (4)
(b)	Use of $\cos 2t = 1 - 2\sin^2 t$	M1
	$\cos 2t = \dfrac{x}{2}$, $\sin t = \dfrac{y}{6}$	
	$\dfrac{x}{2} = 1 - 2\left(\dfrac{y}{6}\right)^2$	M1
	Leading to $y = \sqrt{(18 - 9x)}$ $\left(= 3\sqrt{(2 - x)}\right)$ cao	A1
	$-2 \le x \le 2$ $k = 2$	B1 (4)
(c)	$0 \le f(x) \le 6$ either $0 \le f(x)$ or $f(x) \le 6$	B1
	Fully correct. Accept $0 \le y \le 6$, $[0, 6]$	B1 (2)
		[10]
	Alternatives to (a) *where the parameter is eliminated*	
	① $y = (18 - 9x)^{\frac{1}{2}}$	
	$\dfrac{dy}{dx} = \dfrac{1}{2}(18 - 9x)^{-\frac{1}{2}} \times (-9)$	B1
	At $t = \dfrac{\pi}{3}$, $x = \cos\dfrac{2\pi}{3} = -1$	B1
	$\dfrac{dy}{dx} = \dfrac{1}{2} \times \dfrac{1}{\sqrt{(27)}} \times -9 = -\dfrac{\sqrt{3}}{2}$	M1 A1 (4)
	② $y^2 = 18 - 9x$	
	$2y\dfrac{dy}{dx} = -9$	B1
	At $t = \dfrac{\pi}{3}$, $y = 6\sin\dfrac{\pi}{3} = 3\sqrt{3}$	B1
	$\dfrac{dy}{dx} = -\dfrac{9}{2 \times 3\sqrt{3}} = -\dfrac{\sqrt{3}}{2}$	M1 A1 (4)

Question Number	Scheme	Marks
Q6 (a)	$\int \sqrt{(5-x)}\,\mathrm{d}x = \int (5-x)^{\frac{1}{2}}\,\mathrm{d}x = \frac{(5-x)^{\frac{3}{2}}}{-\frac{3}{2}} \quad (+C)$	M1 A1 (2)
	$\left(= -\frac{2}{3}(5-x)^{\frac{3}{2}} + C\right)$	
(b)	(i) $\int (x-1)\sqrt{(5-x)}\,\mathrm{d}x = -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \frac{2}{3}\int (5-x)^{\frac{3}{2}}\,\mathrm{d}x$	M1 A1ft
	$= \quad \ldots \quad + \frac{2}{3} \times \frac{(5-x)^{\frac{5}{2}}}{-\frac{5}{2}} \quad (+C)$	M1
	$= -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}} \quad (+C)$	A1 (4)
	(ii) $\left[-\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}}\right]_1^5 = (0-0) - \left(0 - \frac{4}{15} \times 4^{\frac{5}{2}}\right)$	
	$= \frac{128}{15}\left(= 8\frac{8}{15} \approx 8.53\right)$ awrt 8.53	M1 A1 (2)
		[8]
	Alternatives for (b) *and* (c)	
	(b) $u^2 = 5-x \Rightarrow 2u\frac{\mathrm{d}u}{\mathrm{d}x} = -1 \quad \left(\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u} = -2u\right)$	
	$\int (x-1)\sqrt{(5-x)}\,\mathrm{d}x = \int (4-u^2)u\frac{\mathrm{d}x}{\mathrm{d}u}\,\mathrm{d}u = \int (4-u^2)u(-2u)\,\mathrm{d}u$	M1 A1
	$= \int (2u^4 - 8u^2)\,\mathrm{d}u = \frac{2}{5}u^5 - \frac{8}{3}u^3 \quad (+C)$	M1
	$= \frac{2}{5}(5-x)^{\frac{5}{2}} - \frac{8}{3}(5-x)^{\frac{3}{2}} \quad (+C)$	A1
	(c) $x=1 \Rightarrow u=2, \quad x=5 \Rightarrow u=0$	
	$\left[\frac{2}{5}u^5 - \frac{8}{3}u^3\right]_2^0 = (0-0) - \left(\frac{64}{5} - \frac{64}{3}\right)$	M1
	$= \frac{128}{15}\left(= 8\frac{8}{15} \approx 8.53\right)$ awrt 8.53	A1 (2)

Question Number	Scheme	Marks
Q7 (a)	$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}10\\14\\-4\end{pmatrix}-\begin{pmatrix}8\\13\\-2\end{pmatrix}=\begin{pmatrix}2\\1\\-2\end{pmatrix}$ or $\overrightarrow{BA}=\begin{pmatrix}-2\\-1\\2\end{pmatrix}$	M1
	$\mathbf{r}=\begin{pmatrix}8\\13\\-2\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ *or* $\mathbf{r}=\begin{pmatrix}10\\14\\-4\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ accept equivalents	M1 A1ft (3)
(b)	$\overrightarrow{CB}=\overrightarrow{OB}-\overrightarrow{OC}=\begin{pmatrix}10\\14\\-4\end{pmatrix}-\begin{pmatrix}9\\9\\6\end{pmatrix}=\begin{pmatrix}1\\5\\-10\end{pmatrix}$ or $\overrightarrow{BC}=\begin{pmatrix}-1\\-5\\10\end{pmatrix}$	
	$CB=\sqrt{\left(1^2+5^2+(-10)^2\right)}=\sqrt{(126)}\quad(=3\sqrt{14}\approx 11.2)$ awrt 11.2	M1 A1 (2)
(c)	$\overrightarrow{CB}.\overrightarrow{AB}=\lvert\overrightarrow{CB}\rvert\lvert\overrightarrow{AB}\rvert\cos\theta$	
	$(\pm)(2+5+20)=\sqrt{126}\sqrt{9}\cos\theta$	M1 A1
	$\cos\theta=\dfrac{3}{\sqrt{14}}\ \Rightarrow\ \theta\approx 36.7°$ awrt 36.7°	A1 (3)
(d)		
	$\dfrac{d}{\sqrt{126}}=\sin\theta$	M1 A1ft
	$d=3\sqrt{5}(\approx 6.7)$ awrt 6.7	A1 (3)
(e)	$BX^2=BC^2-d^2=126-45=81$	M1
	! $CBX=\frac{1}{2}\times BX\times d=\frac{1}{2}\times 9\times 3\sqrt{5}=\dfrac{27\sqrt{5}}{2}\ (\approx 30.2)$ awrt 30.1 or 30.2	M1 A1 (3)
		[14]
	Alternative for (e)	
	! $CBX=\frac{1}{2}\times d\times BC\sin\angle XCB$	M1
	$=\frac{1}{2}\times 3\sqrt{5}\times\sqrt{126}\sin(90-36.7)°$ sine of correct angle	M1
	≈ 30.2 $\quad\dfrac{27\sqrt{5}}{2}$, awrt 30.1 or 30.2	A1 (3)

Question Number	Scheme	Marks
Q8 (a)	$\int \sin^2\theta \,d\theta = \frac{1}{2}\int (1-\cos 2\theta)\,d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \quad (+C)$	M1 A1 (2)
(b)	$x = \tan\theta \Rightarrow \frac{dx}{d\theta} = \sec^2\theta$	
	$\pi\int y^2\,dx = \pi\int y^2 \frac{dx}{d\theta}\,d\theta = \pi\int (2\sin 2\theta)^2 \sec^2\theta\,d\theta$	M1 A1
	$= \pi\int \frac{(2\times 2\sin\theta\cos\theta)^2}{\cos^2\theta}\,d\theta$	M1
	$= 16\pi\int \sin^2\theta\,d\theta \qquad k = 16\pi$	A1
	$x = 0 \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0, \quad x = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$	B1 (5)
	$\left(V = 16\pi\int_0^{\frac{\pi}{6}} \sin^2\theta\,d\theta\right)$	
(c)	$V = 16\pi\left[\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{6}}$	M1
	$= 16\pi\left[\left(\frac{\pi}{12} - \frac{1}{4}\sin\frac{\pi}{3}\right) - (0-0)\right]$ Use of correct limits	M1
	$= 16\pi\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) = \frac{4}{3}\pi^2 - 2\pi\sqrt{3} \qquad p = \frac{4}{3},\ q = -2$	A1 (3)
		[10]

June 2009
6667 Further Pure Mathematics FP1 (new)
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)		B1 (1)
(b)	$\lvert z_1 \rvert = \sqrt{2^2 + (-1)^2} = \sqrt{5}$ (or awrt 2.24)	M1 A1 (2)
(c)	$\alpha = \arctan\left(\frac{1}{2}\right)$ or $\arctan\left(-\frac{1}{2}\right)$	M1
	$\arg z_1 = -0.46$ or 5.82 (awrt) (answer in degrees is A0 unless followed by correct conversion)	A1 (2)
(d)	$\frac{-8+9\mathrm{i}}{2-\mathrm{i}} \times \frac{2+\mathrm{i}}{2+\mathrm{i}}$	M1
	$= \frac{-16-8\mathrm{i}+18\mathrm{i}-9}{5} = -5+2\mathrm{i}$ i.e. a = -5 and b =2 or $-\frac{2}{5}a$	A1 A1ft (3) **[8]**
	Alternative method to part (d)	
	$-8+9\mathrm{i} = (2-i)(a+b\mathrm{i})$, and so $2a+b=-8$ and $2b-a=9$ and attempt to solve as far as equation in one variable	M1
	So a = -5 and $b = 2$	A1 A1cao
Notes	(a) B1 needs both complex numbers as either points or vectors, in correct quadrants and with 'reasonably correct' relative scale	
	(b) M1 Attempt at Pythagoras to find modulus of either complex number	
	A1 condone correct answer even if negative sign not seen in (-1) term	
	A0 for $\pm\sqrt{5}$	
	(c) arctan 2 is M0 unless followed by $\boxed{\frac{3\pi}{2} + \arctan 2}$ or $\boxed{\frac{\pi}{2} - \arctan 2}$ Need to be clear that argz = - 0.46 or 5.82 for A1	
	(d) M1 Multiply numerator and denominator by conjugate of their denominator	
	A1 for –5 and A1 for 2i (should be simplified)	
	Alternative scheme for (d) Allow slips in working for first M1	

Question Number	Scheme	Marks
Q2 (a)	$r(r+1)(r+3) = r^3 + 4r^2 + 3r$, so **use** $\sum r^3 + 4\sum r^2 + 3\sum r$	M1
	$= \frac{1}{4}n^2(n+1)^2 + 4\left(\frac{1}{6}n(n+1)(2n+1)\right) + 3\left(\frac{1}{2}n(n+1)\right)$	A1 A1
	$= \frac{1}{12}n(n+1)\{3n(n+1) + 8(2n+1) + 18\}$ or $= \frac{1}{12}n\{3n^3 + 22n^2 + 45n + 26\}$	
	or $= \frac{1}{12}(n+1)\{3n^3 + 19n^2 + 26n\}$	M1 A1
	$= \frac{1}{12}n(n+1)\{3n^2 + 19n + 26\} = \frac{1}{12}n(n+1)(n+2)(3n+13)$ $(k = 13)$	M1 A1cao (7)
(b)	$\sum_{21}^{40} = \sum_{1}^{40} - \sum_{1}^{20}$	M1
	$= \frac{1}{12}(40\times41\times42\times133) - \frac{1}{12}(20\times21\times22\times73) = 763420 - 56210 = 707210$	A1 cao (2) **[9]**
Notes	(a) M1 expand and must start to use at least one standard formula First 2 A marks: One wrong term A1 A0, two wrong terms A0 A0. M1: Take out factor $kn(n+1)$ or kn or $k(n+1)$ directly or from quartic A1: See scheme (cubics must be simplified) M1: Complete method including a quadratic factor and attempt to factorise it A1 Completely correct work. Just gives $k = 13$, no working is **0** marks for the question. **Alternative method** Expands $(n+1)(n+2)(3n+k)$ and confirms that it equals $\{3n^3 + 22n^2 + 45n + 26\}$ together with statement $k = 13$ can earn last **M1A1** The previous **M1A1** can be implied if they are using a quartic. **(b)** M 1 is for substituting 40 and 20 into their **answer** to (a) and subtracting. (NB not 40 and 21) Adding terms is M0A0 as the question said "Hence"	

Question Number	Scheme	Marks
Q3 (a)	$x^2 + 4 = 0 \quad \Rightarrow \quad x = ki, \quad x = \pm 2i$	M1, A1
	Solving 3-term quadratic	M1
	$x = \dfrac{-8 \pm \sqrt{64 - 100}}{2} \quad = -4 + 3i$ and $-4 - 3i$	A1 A1ft
		(5)
(b)	$2i + (-2i) + (-4 + 3i) + (-4 - 3i) = -8$	M1 A1cso
		(2)
		[7]
	Alternative method : Expands f(x) as quartic and chooses $\pm$ coefficient of x^3	M1
	-8	A1 cso
Notes	(a) Just $x = 2i$ is M1 A0 $x = \pm 2$ is M0A0 M1 for solving quadratic follows usual conventions, then A1 for a correct root (simplified as here) and A1ft for conjugate of first answer. Accept correct answers with no working here. Do not give accuracy marks for factors unless followed by roots. (b) M1 for adding four roots of which at least two are complex conjugates and getting a real answer. A1 for –8 following **correct** roots or the alternative method. If any incorrect working in part (a) this A mark will be A0	

Question Number	Scheme	Marks
Q4 (a)	$f(2.2) = 2.2^3 - 2.2^2 - 6 \quad (= -0.192)$ $f(2.3) = 2.3^3 - 2.3^2 - 6 \quad (= 0.877)$ Change of sign $\Rightarrow$ Root need numerical values correct (to 1 s.f.).	M1 A1 (2)
(b)	$f'(x) = 3x^2 - 2x$ $f'(2.2) = 10.12$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.2 - \frac{-0.192}{10.12}$ $= 2.219$	B1 B1 M1 A1ft A1cao (5)
(c)	$\frac{\alpha - 2.2}{\pm'0.192'} = \frac{2.3-\alpha}{\pm'0.877'}$ (or equivalent such as $\frac{k}{\pm'0.192'} = \frac{0.1-k}{\pm'0.877'}$.) $\alpha(0.877 + 0.192) = 2.3 \times 0.192 + 2.2 \times 0.877$ or $k(0.877 + 0.192) = 0.1 \times 0.192$, where $\alpha = 2.2 + k$ so $\alpha \approx 2.218$ (2.21796…) (Allow awrt)	M1 A1 A1 (3) **[10]**
Alternative	Uses equation of line joining (2.2, -0.192) to (2.3, 0.877) and substitutes $y = 0$ $y + 0.192 = \frac{0.192 + 0.877}{0.1}(x - 2.2)$ and $y = 0$, so $\alpha \approx 2.218$ or awrt as before (NB Gradient = 10.69)	M1 A1, A1
Notes	(a) M1 for attempt at f(2.2) and f(2.3) A1 need indication that there is a change of sign – (could be –0.19<0, 0.88>0) and need conclusion. (These marks may be awarded in other parts of the question if not done in part (a)) (b) B1 for seeing correct derivative (but may be implied by later correct work) B1 for seeing 10.12 or this may be implied by later work M1 Attempt Newton-Raphson with their values A1ft may be implied by the following answer (but does not require an evaluation) Final A1 must 2.219 exactly as shown. So answer of 2.21897 would get 4/5 If done twice ignore second attempt (c) M1 Attempt at ratio with their values of ± f(2.2) and ± f(2.3). N.B. If you see $0.192 - \alpha$ or $0.877 - \alpha$ in the fraction then this is M0 A1 correct linear expression and definition of variable if not α (may be implied by final correct answer- does not need 3 dp accuracy) A1 for awrt 2.218 If done twice ignore second attempt	

Question Number	Scheme	Marks
Q5 (a)	$\mathbf{R}^2 = \begin{pmatrix} a^2+2a & 2a+2b \\ a^2+ab & 2a+b^2 \end{pmatrix}$	M1 A1 A1 (3)
(b)	Puts their $a^2+2a = 15$ or their $2a+b^2 = 15$ or their $(a^2+2a)(2a+b^2)-(a^2+ab)(2a+2b) = 225$ (or to 15) ,	M1,
	Puts their $a^2+ab=0$ or their $2a+2b=0$	M1
	Solve to find either a or b	M1
	$a=3, \quad b=-3$	A1, A1 (5) **[8]**
Alternative for (b)	Uses $\mathbf{R}^2 \times$ column vector = $15\times$ column vector, and equates rows to give two equations in a and b only Solves to find either a or b as above method	M1, M1 M1 A1 A1
Notes	(a) 1 term correct: M1 A0 A0 2 or 3 terms correct: M1 A1 A0 (b) M1 M1 as described in scheme (In the alternative scheme column vector can be general or specific for first M1 but must be specific for 2nd M1) M1 requires solving equations to find a and/or b (though checking that correct answer satisfies the equations will earn this mark) This mark can be given independently of the first two method marks. So solving $\mathbf{M}^2 = 15\mathbf{M}$ for example gives M0M0M1A0A0 in part (b) Also putting leading diagonal = 0 and other diagonal = 15 is M0M0M1A0A0 (No possible solutions as a >0) A1 A1 for correct answers only **Any Extra answers given**, e.g. a = -5 and b = 5 or wrong answers – **deduct last A1 awarded** So the two sets of answers would be A1 A0 Just the answer . a = -5 and b = 5 is A0 A0 Stopping at two values for a or for b – no attempt at other is A0A0 Answer with no working at all is 0 marks	

Question Number	Scheme	Marks
Q6 (a)	$y^2 = (8t)^2 = 64t^2$ and $16x = 16 \times 4t^2 = 64t^2$	B1
	Or identifies that $a = 4$ and uses general coordinates $(at^2, 2at)$	(1)
(b)	$(4, 0)$	B1
		(1)
(c)	$y = 4x^{\frac{1}{2}}$ $\quad \frac{dy}{dx} = 2x^{-\frac{1}{2}}$	B1
	Replaces x by $4t^2$ to give **gradient** $\quad [2(4t^2)^{-\frac{1}{2}} = \frac{2}{2t} = \frac{1}{t}]$	M1,
	Uses Gradient of normal is $-\frac{1}{\text{gradient of curve}}$ $\quad [-t]$	M1
	$y - 8t = -t(x - 4t^2) \quad \Rightarrow \quad y + tx = 8t + 4t^3$ (*)	M1 A1cso
		(5)
(d)	At N, $y = 0$, so $x = 8 + 4t^2$ or $\frac{8t + 4t^3}{t}$	B1
	Base $SN = (8 + 4t^2) - 4 \quad (= 4 + 4t^2)$	B1ft
	Area of $\Delta\, PSN = \frac{1}{2}(4 + 4t^2)(8t) = 16t(1 + t^2)$ or $16t + 16t^3$ for $t > 0$	M1 A1
	{Also Area of $\Delta\, PSN = \frac{1}{2}(4 + 4t^2)(-8t) = -16t(1 + t^2)$ for $t < 0$ } *this is not required*	(4) **[11]**
	<u>Alternatives:</u>	
	(c) $\frac{dx}{dt} = 8t$ and $\frac{dy}{dt} = 8$ $\quad$ B1	
	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{1}{t}$ $\quad$ M1 , then as in main scheme.	
	(c) $2y\frac{dy}{dx} = 16$ $\quad$ B1 (or uses $x = \frac{y^2}{8}$ to give $\frac{dx}{dy} = \frac{2y}{8}$)	
	$\frac{dy}{dx} = \frac{8}{y} = \frac{8}{8t} = \frac{1}{t}$ $\quad$ M1 , then as in main scheme.	
Notes	(c) Second M1 – need not be function of t Third M1 requires linear equation (not fraction) and should include the parameter t but could be given for equation of tangent (So tangent equation loses 2 marks only and could gain B1M1M0M1A0) (d) Second B1 does not require simplification and may be a constant rather than an expression in t. M1 needs correct area of triangle formula using ½ 'their SN' $\times 8t$ Or may use two triangles in which case need $(4t^2 - 4)$ and $(4t^2 + 8 - 4t^2)$ for B1ft Then Area of $\Delta\, PSN = \frac{1}{2}(4t^2 - 4)(8t) + \frac{1}{2}(4t^2 + 8 - 4t^2)(8t) = 16t(1 + t^2)$ or $16t + 16t^3$	

Question Number	Scheme	Marks
Q7 (a)	Use $4a-(-2\times -1)=0 \quad \Rightarrow \quad a,=\frac{1}{2}$	M1, A1 (2)
(b)	Determinant: $(3\times 4)-(-2\times -1)=10 \quad (\Delta)$	M1
	$\mathbf{B}^{-1}=\frac{1}{10}\begin{pmatrix}4 & 2\\ 1 & 3\end{pmatrix}$	M1 A1cso (3)
(c)	$\frac{1}{10}\begin{pmatrix}4 & 2\\ 1 & 3\end{pmatrix}\begin{pmatrix}k-6\\ 3k+12\end{pmatrix}, \quad =\frac{1}{10}\begin{pmatrix}4(k-6)+2(3k+12)\\ (k-6)+3(3k+12)\end{pmatrix}$	M1, A1ft
	$\begin{pmatrix}k\\ k+3\end{pmatrix}$ Lies on $y=x+3$	A1 (3) **[8]**
Notes	Alternatives: (c) $\begin{pmatrix}3 & -2\\ -1 & 4\end{pmatrix}\begin{pmatrix}x\\ x+3\end{pmatrix}, \quad =\begin{pmatrix}3x-2(x+3)\\ -x+4(x+3)\end{pmatrix}$, $=\begin{pmatrix}x-6\\ 3x+12\end{pmatrix}$, which was of the form $(k - 6, 3k + 12)$ Or $\begin{pmatrix}3 & -2\\ -1 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}, \quad =\begin{pmatrix}3x-2y\\ -x+4y\end{pmatrix}=\begin{pmatrix}k-6\\ 3k+12\end{pmatrix}$, and solves simultaneous equations Both equations correct and eliminate one letter to get $x = k$ or $y = k + 3$ or $10x-10y=-30$ or equivalent. Completely correct work (to $x = k$ and $y = k + 3$), and conclusion lies on $y=x+3$	M1, A1, A1 M1 A1 A1
	(a) Allow sign slips for first M1 (b) Allow sign slip for determinant for first M1 (This mark may be awarded for 1/10 appearing in inverse matrix.) Second M1 is for correctly treating the 2 by 2 matrix, ie for $\begin{pmatrix}4 & 2\\ 1 & 3\end{pmatrix}$ Watch out for determinant (3 + 4) – (-1 + -2) = 10 – M0 then final answer is A0 (c) M1 for multiplying matrix by appropriate column vector A1 correct work (ft wrong determinant) A1 for conclusion	

Question Number	Scheme	Marks
Q8 (a)	$f(1) = 5+8+3 = 16$, (which is divisible by 4). ($\therefore$ True for n = 1).	B1
	Using the formula to write down f(k + 1), $f(k+1) = 5^{k+1} + 8(k+1) + 3$	M1 A1
	$f(k+1) - f(k) = 5^{k+1} + 8(k+1) + 3 - 5^k - 8k - 3$	M1
	$= 5(5^k) + 8k + 8 + 3 - 5^k - 8k - 3 = 4(5^k) + 8$	A1
	$f(k+1) = 4(5^k + 2) + f(k)$, **which is divisible by** 4	A1ft
	$\therefore$ True for $n = k + 1$ **if** true for $n = k$. True for n = 1, $\therefore$ true for all n.	A1cso (7)
(b)	For n = 1, $\begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^1$ ($\therefore$ True for n = 1.)	B1
	$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$	M1 A1 A1
	$= \begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$	M1 A1
	$\therefore$ True for $n = k + 1$ **if** true for $n = k$. **True for n** = 1, $\therefore$ **true for all n**	A1 cso (7) **[14]**
(a) Alternative for 2nd M:	$f(k+1) = 5(5^k) + 8k + 8 + 3$ M1 $= 4(5^k) + 8 + (5^k + 8k + 3)$ A1 or $= 5(5^k + 8k + 3) - 32k - 4$ $= 4(5^k + 2) + f(k)$, or $= 5f(k) - 4(8k+1)$ which is divisible by 4 A1 (or similar methods)	

Notes

(a) B1 Correct values of 16 or 4 for n = 1 or for n = 0 (Accept "is a multiple of")
M1 Using the formula to write down f(k + 1) A1 Correct expression of f(k+1) (or for f(n +1)
M1 Start method to connect f(k+1) with f(k) as shown
A1 correct working toward multiples of 4, A1 ft result including f(k +1) as subject, A1cso conclusion

(b) B1 correct statement for n = 1 or n = 0
First M1: Set up product of two appropriate matrices – product can be either way round
A1 A0 for one or two slips in simplified result
A1 A1 all correct simplified
A0 A0 more than two slips
M1: States in terms of (k + 1)
A1 Correct statement A1 for induction conclusion

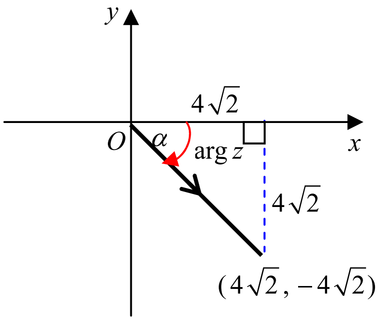
Part (b) Alternative

May write $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$. Then may or may not complete the proof.
This can be awarded the second M (substituting k + 1)and following A (simplification) in part (b).
The first three marks are awarded as before . Concluding that they have reached the same matrix and therefore a result will then be part of final A1 cso but also need other statements as in the first method.

June 2009
6668 Further Pure Mathematics FP2 (new)
Mark Scheme

Question Number	Scheme		Marks
Q1 (a)	$\frac{1}{r(r+2)} = \underline{\frac{1}{2r} - \frac{1}{2(r+2)}}$	$\underline{\frac{1}{2r} - \frac{1}{2(r+2)}}$	B1 **aef**
			(1)
(b)	$\sum_{r=1}^{n} \frac{4}{r(r+2)} = \sum_{r=1}^{n} \left(\frac{2}{r} - \frac{2}{r+2}\right)$		
	$= \left(\underline{\frac{2}{1}} - \frac{2}{3}\right) + \left(\underline{\frac{2}{2}} - \frac{2}{4}\right) + \;\dots\dots$ $\dots\dots\dots\dots + \left(\frac{2}{n-1} - \underline{\frac{2}{n+1}}\right) + \left(\frac{2}{n} - \underline{\frac{2}{n+2}}\right)$	List the first two terms and the last two terms	M1
	$= \frac{2}{1} + \frac{2}{2}\,;- \frac{2}{n+1} - \frac{2}{n+2}$	Includes the first two underlined terms and includes the final two underlined terms.	M1
		$\frac{2}{1} + \frac{2}{2} - \frac{2}{n+1} - \frac{2}{n+2}$	A1
	$= 3 - \frac{2}{n+1} - \frac{2}{n+2}$		
	$= \frac{3(n+1)(n+2) - 2(n+2) - 2(n+1)}{(n+1)(n+2)}$ $= \frac{3n^2 + 9n + 6 - 2n - 4 - 2n - 2}{(n+1)(n+2)}$	Attempt to combine to an at least 3 term fraction to a single fraction and an attempt to take out the brackets from their numerator.	M1
	$= \frac{3n^2 + 5n}{(n+1)(n+2)}$		
	$= \frac{n(3n+5)}{(n+1)(n+2)}$	Correct Result	A1 **cso AG**
			(5)
			[6]

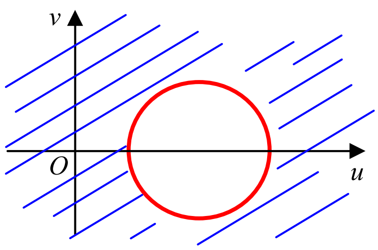
edexcel

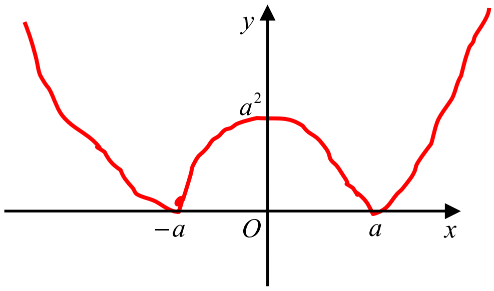
Question Number	Scheme		Marks	
Q2 (a)	$z^3 = 4\sqrt{2} - 4\sqrt{2}\,\mathrm{i}\ ,\ \ -\pi < \theta \leqslant \pi$			
				
	$r = \sqrt{(4\sqrt{2})^2 + (-4\sqrt{2})^2} = \sqrt{32+32} = \sqrt{64} = 8$ $\theta = -\tan^{-1}\left(\frac{4\sqrt{2}}{4\sqrt{2}}\right) = -\frac{\pi}{4}$	A valid attempt to find the modulus and argument of $4\sqrt{2} - 4\sqrt{2}\,\mathrm{i}$.	M1	
	$z^3 = 8\left(\cos\left(-\frac{\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{\pi}{4}\right)\right)$			
	So, $z = (8)^{\frac{1}{3}}\left(\cos\left(\frac{-\frac{\pi}{4}}{3}\right) + \mathrm{i}\sin\left(\frac{-\frac{\pi}{4}}{3}\right)\right)$	Taking the cube root of the modulus and dividing the argument by 3.	M1	
	$\Rightarrow\ z = 2\left(\cos\left(-\frac{\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{\pi}{12}\right)\right)$	$2\left(\cos\left(-\frac{\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{\pi}{12}\right)\right)$	A1	
	Also, $z^3 = 8\left(\cos\left(\frac{7\pi}{4}\right) + \mathrm{i}\sin\left(\frac{7\pi}{4}\right)\right)$ or $z^3 = 8\left(\cos\left(-\frac{9\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{9\pi}{4}\right)\right)$	Adding or subtracting 2π to the argument for z^3 in order to find other roots.	M1	
	$\Rightarrow\ z = 2\left(\cos\frac{7\pi}{12} + \mathrm{i}\sin\frac{7\pi}{12}\right)$	Any one of the final two roots	A1	
	and $z = 2\left(\cos\left(\frac{-3\pi}{4}\right) + \mathrm{i}\sin\left(\frac{-3\pi}{4}\right)\right)$	Both of the final two roots.	A1	
				[6]
	Special Case 1: Award SC: M1M1A1M1A0A0 for ALL three of $2\left(\cos\frac{\pi}{12} + \mathrm{i}\sin\frac{\pi}{12}\right)$, $2\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$ and $2\left(\cos\left(\frac{-7\pi}{12}\right) + \mathrm{i}\sin\left(\frac{-7\pi}{12}\right)\right)$.			
	Special Case 2: If r is incorrect (and not equal to 8) and candidate states the brackets $(\qquad)$ correctly then give the first accuracy mark ONLY where this is applicable.			

Question Number	Scheme		Marks
Q3	$\sin x\dfrac{dy}{dx} - y\cos x = \sin 2x\sin x$		
	$\dfrac{dy}{dx} - \dfrac{y\cos x}{\sin x} = \dfrac{\sin 2x\sin x}{\sin x}$	An attempt to divide every term in the differential equation by $\sin x$. Can be implied.	M1
	$\dfrac{dy}{dx} - \dfrac{y\cos x}{\sin x} = \sin 2x$		
	Integrating factor $= e^{\int -\frac{\cos x}{\sin x}\,dx} = e^{-\ln\sin x}$	$e^{\int \pm\frac{\cos x}{\sin x}(dx)}$ or $e^{\int \pm\text{their } P(x)\,(dx)}$	dM1
		$e^{-\ln\sin x}$ or $e^{\ln\operatorname{cosec} x}$	A1 aef
	$= \dfrac{1}{\sin x}$	$\dfrac{1}{\sin x}$ or $(\sin x)^{-1}$ or $\operatorname{cosec} x$	A1 aef
	$\left(\dfrac{1}{\sin x}\right)\dfrac{dy}{dx} - \dfrac{y\cos x}{\sin^2 x} = \dfrac{\sin 2x}{\sin x}$		
	$\dfrac{d}{dx}\left(\dfrac{y}{\sin x}\right) = \sin 2x \times \dfrac{1}{\sin x}$	$\dfrac{d}{dx}(y \times \text{their I.F.}) = \sin 2x \times \text{their I.F}$	M1
	$\dfrac{d}{dx}\left(\dfrac{y}{\sin x}\right) = 2\cos x$	$\dfrac{d}{dx}\left(\dfrac{y}{\sin x}\right) = 2\cos x$ or $\dfrac{y}{\sin x} = \int 2\cos x\,(dx)$	A1
	$\dfrac{y}{\sin x} = \int 2\cos x\,dx$		
	$\dfrac{y}{\sin x} = 2\sin x + K$	A credible attempt to integrate the RHS with/without $+K$	dddM1
	$y = 2\sin^2 x + K\sin x$	$y = 2\sin^2 x + K\sin x$	A1 cao
			[8]

Question Number	Scheme		Marks
Q4	$A = \frac{1}{2}\int_0^{2\pi}(a + 3\cos\theta)^2\,d\theta$	Applies $\frac{1}{2}\int_0^{2\pi} r^2\,(d\theta)$ with correct limits. Ignore $d\theta$.	B1
	$(a + 3\cos\theta)^2 = a^2 + 6a\cos\theta + 9\cos^2\theta$		
	$= \underline{a^2 + 6a\cos\theta + 9\left(\frac{1+\cos 2\theta}{2}\right)}$	$\cos^2\theta = \frac{\pm 1 \pm \cos 2\theta}{2}$ Correct underlined expression.	M1 A1
	$A = \frac{1}{2}\int_0^{2\pi}\left(a^2 + 6a\cos\theta + \frac{9}{2} + \frac{9}{2}\cos 2\theta\right)d\theta$		
	$= \left(\frac{1}{2}\right)\left[a^2\theta + 6a\sin\theta + \frac{9}{2}\theta + \frac{9}{4}\sin 2\theta\right]_0^{2\pi}$	Integrated expression with at least 3 out of 4 terms of the form $\pm A\theta \pm B\sin\theta \pm C\theta \pm D\sin 2\theta$. Ignore the $\frac{1}{2}$. Ignore limits. $a^2\theta + 6a\sin\theta +$ correct ft integration. Ignore the $\frac{1}{2}$. Ignore limits.	M1* A1 ft
	$= \frac{1}{2}\left[\left(2\pi a^2 + 0 + 9\pi + 0\right) - (0)\right]$		
	$= \pi a^2 + \frac{9\pi}{2}$	$\pi a^2 + \frac{9\pi}{2}$	A1
	Hence, $\pi a^2 + \frac{9\pi}{2} = \frac{107}{2}\pi$	Integrated expression equal to $\frac{107}{2}\pi$.	dM1*
	$a^2 + \frac{9}{2} = \frac{107}{2}$		
	$a^2 = 49$		
	As $a > 0$, $a = 7$	$a = 7$	A1 cso **[8]**
	Some candidates may achieve $a = 7$ from incorrect working. Such candidates will not get full marks		

Question Number	Scheme		Marks
Q5	$y = \sec^2 x = (\sec x)^2$		
(a)	$\frac{dy}{dx} = 2(\sec x)^1(\sec x \tan x) = 2\sec^2 x \tan x$	Either $2(\sec x)^1(\sec x \tan x)$ or $2\sec^2 x \tan x$	B1 aef
	Apply product rule: $\left\{\begin{matrix} u = 2\sec^2 x & v = \tan x \\ \frac{du}{dx} = 4\sec^2 x \tan x & \frac{dv}{dx} = \sec^2 x \end{matrix}\right\}$		
	$\frac{d^2y}{dx^2} = 4\sec^2 x \tan^2 x + 2\sec^4 x$	Two terms added with one of either $A\sec^2 x \tan^2 x$ or $B\sec^4 x$ in the correct form.	M1
		Correct differentiation	A1
	$= 4\sec^2 x(\sec^2 x - 1) + 2\sec^4 x$		
	Hence, $\frac{d^2y}{dx^2} = 6\sec^4 x - 4\sec^2 x$	Applies $\tan^2 x = \sec^2 x - 1$ leading to the correct result.	A1 **AG**
			(4)
(b)	$y_{\frac{\pi}{4}} = \left(\sqrt{2}\right)^2 = \underline{2}$, $\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = 2\left(\sqrt{2}\right)^2(1) = \underline{4}$	Both $y_{\frac{\pi}{4}} = \underline{2}$ and $\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = \underline{4}$	B1
	$\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{4}} = 6\left(\sqrt{2}\right)^4 - 4\left(\sqrt{2}\right)^2 = 24 - 8 = 16$	Attempts to substitute $x = \frac{\pi}{4}$ into both terms in the expression for $\frac{d^2y}{dx^2}$.	M1
	$\frac{d^3y}{dx^3} = 24\sec^3 x(\sec x \tan x) - 8\sec x(\sec x \tan x)$	Two terms differentiated with either $24\sec^4 x \tan x$ or $-8\sec^2 x \tan x$ being correct	M1
	$= 24\sec^4 x \tan x - 8\sec^2 x \tan x$		
	$\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{4}} = 24\left(\sqrt{2}\right)^4(1) - 8\left(\sqrt{2}\right)^2(1) = 96 - 16 = 80$	$\left(\frac{d^3y}{dx^3}\right)_{\frac{\pi}{4}} = \underline{80}$	B1
	$\sec x \approx 2 + 4\left(x - \frac{\pi}{4}\right) + \frac{16}{2}\left(x - \frac{\pi}{4}\right)^2 + \frac{80}{6}\left(x - \frac{\pi}{4}\right)^3 + \ldots$	Applies a Taylor expansion with at least 3 out of 4 terms ft correctly.	M1
		Correct Taylor series expansion.	A1
			(6)
	$\left\{\sec x \approx 2 + 4\left(x - \frac{\pi}{4}\right) + 8\left(x - \frac{\pi}{4}\right)^2 + \frac{40}{3}\left(x - \frac{\pi}{4}\right)^3 + \ldots\right\}$		
			[10]

Question Number	Scheme		Marks
Q6	$w = \dfrac{z}{z+\mathrm{i}}, \quad z = -\mathrm{i}$		
(a)	$w(z+\mathrm{i}) = z \Rightarrow wz + \mathrm{i}w = z \Rightarrow \mathrm{i}w = z - wz$	Complete method of rearranging to make z the subject.	M1
	$\Rightarrow \mathrm{i}w = z(1-w) \Rightarrow z = \dfrac{\mathrm{i}w}{(1-w)}$	$z = \dfrac{\mathrm{i}w}{(1-w)}$	A1 **aef**
	$\|z\| = 3 \Rightarrow \left\|\dfrac{\mathrm{i}w}{1-w}\right\| = 3$	Putting $\|z$ in terms of their $w\| = 3$	dM1
	$\left\{\begin{array}{l} \|\mathrm{i}w\| = 3\|1-w\| \Rightarrow \|w\| = 3\|w-1\| \Rightarrow \|w\|^2 = 9\|w-1\|^2 \\ \Rightarrow \|u+\mathrm{i}v\|^2 = 9\|u+\mathrm{i}v-1\|^2 \end{array}\right\}$		
	$\Rightarrow u^2 + v^2 = 9\left[(u-1)^2 + v^2\right]$	Applies $w = u + \mathrm{i}v$, and uses Pythagoras correctly to get an equation in terms of u and v without any i's.	ddM1
		Correct equation.	A1
	$\left\{\begin{array}{l} \Rightarrow u^2 + v^2 = 9u^2 - 18u + 9 + 9v^2 \\ \Rightarrow 0 = 8u^2 - 18u + 8v^2 + 9 \end{array}\right\}$		
	$\Rightarrow 0 = u^2 - \frac{9}{4}u + v^2 + \frac{9}{8}$	Simplifies down to $u^2 + v^2 \pm \alpha u \pm \beta v \pm \delta = 0.$	dddM1
	$\Rightarrow \left(u - \frac{9}{8}\right)^2 - \frac{81}{64} + v^2 + \frac{9}{8} = 0$		
	$\Rightarrow \left(u - \frac{9}{8}\right)^2 + v^2 = \frac{9}{64}$		
	{Circle} centre $\left(\frac{9}{8}, 0\right)$, radius $\frac{3}{8}$	One of centre or radius correct.	A1
		Both centre and radius correct.	A1
			(8)
(b)		Circle indicated on the Argand diagram in the correct position in follow through quadrants. Ignore plotted coordinates.	B1ft
		Region outside a circle indicated only.	B1
			(2)
			[10]

Question Number	Scheme		Marks
Q7	$y = \left\|x^2 - a^2\right\|,\ a > 1$		
(a)		Correct Shape. Ignore cusps. Correct coordinates.	B1 B1
			(2)
(b)	$\left\|x^2 - a^2\right\| = a^2 - x\ ,\ a > 1$		
	$\{\|x\| > a\},\quad x^2 - a^2 = a^2 - x$	$x^2 - a^2 = a^2 - x$	M1 aef
	$\Rightarrow\ x^2 + x - 2a^2 = 0$		
	$\Rightarrow\ x = \dfrac{-1 \pm \sqrt{1 - 4(1)(-2a^2)}}{2}$	Applies the quadratic formula or completes the square in order to find the roots.	M1
	$\Rightarrow\ x = \dfrac{-1 \pm \sqrt{1 + 8a^2}}{2}$	Both correct "simplified down" solutions.	A1
	$\{\|x\| < a\},\quad -x^2 + a^2 = a^2 - x$	$-x^2 + a^2 = a^2 - x$ or $x^2 - a^2 = x - a^2$	M1 aef
	$\{\Rightarrow\ x^2 - x = 0 \Rightarrow x(x-1) = 0\}$		
	$\Rightarrow\ x = 0\ ,1$	$x = 0$ $x = 1$	B1 A1
			(6)
(c)	$\left\|x^2 - a^2\right\| > a^2 - x\ ,\ a > 1$		
	$x < \dfrac{-1 - \sqrt{1 + 8a^2}}{2}\quad \{\text{or}\}\quad x > \dfrac{-1 + \sqrt{1 + 8a^2}}{2}$	x is less than their least value x is greater than their maximum value	B1 ft B1 ft
	$\{\text{or}\}\quad 0 < x < 1$	For $\{\|x\| < a\}$, Lowest $< x <$ Highest $0 < x < 1$	M1 A1
			(4)
			[12]

Question Number	Scheme		Marks
Q8	$\dfrac{d^2x}{dt^2} + 5\dfrac{dx}{dt} + 6x = 2e^{-t}\ ,\quad x = 0,\ \dfrac{dx}{dt} = 2$ at $t = 0.$		
(a)	AE, $m^2 + 5m + 6 = 0 \Rightarrow (m+3)(m+2) = 0$ $\Rightarrow m = -3, -2.$		
	So, $x_{CF} = Ae^{-3t} + Be^{-2t}$	$Ae^{m_1 t} + Be^{m_2 t}$, where $m_1 \neq m_2$. $Ae^{-3t} + Be^{-2t}$	M1 A1
	$\left\{ x = ke^{-t} \Rightarrow \dfrac{dx}{dt} = -ke^{-t} \Rightarrow \dfrac{d^2x}{dt^2} = ke^{-t} \right\}$		
	$\Rightarrow ke^{-t} + 5(-ke^{-t}) + 6ke^{-t} = 2e^{-t} \Rightarrow 2ke^{-t} = 2e^{-t}$ $\Rightarrow k = 1$	Substitutes ke^{-t} into the differential equation given in the question. Finds $k = 1$.	M1 A1
	$\left\{ \text{So, } x_{PI} = e^{-t} \right\}$		
	So, $x = Ae^{-3t} + Be^{-2t} + e^{-t}$	their x_{CF} + their x_{PI}	M1*
	$\dfrac{dx}{dt} = = -3Ae^{-3t} - 2Be^{-2t} - e^{-t}$	Finds $\dfrac{dx}{dt}$ by differentiating their x_{CF} and their x_{PI}	dM1*
	$t = 0,\ x = 0 \Rightarrow \quad 0 = A + B + 1$ $t = 0,\ \dfrac{dx}{dt} = 2 \Rightarrow \quad 2 = -3A - 2B - 1$	Applies $t = 0,\ x = 0$ to x and $t = 0,\ \dfrac{dx}{dt} = 2$ to $\dfrac{dx}{dt}$ to form simultaneous equations.	ddM1*
	$\left\{ \begin{array}{l} 2A + 2B = -2 \\ -3A - 2B = 3 \end{array} \right\}$		
	$\Rightarrow A = -1,\ B = 0$		
	So, $x = -e^{-3t} + e^{-t}$	$x = -e^{-3t} + e^{-t}$	A1 **cao** (8)

Question Number	Scheme		Marks
	$x = -e^{-3t} + e^{-t}$		
(b)	$\dfrac{dx}{dt} = 3e^{-3t} - e^{-t} = 0$	Differentiates their x to give $\dfrac{dx}{dt}$ and puts $\dfrac{dx}{dt}$ equal to 0.	M1
	$3 - e^{2t} = 0$	A credible attempt to solve.	dM1*
	$\Rightarrow t = \frac{1}{2}\ln 3$	$t = \frac{1}{2}\ln 3$ or $t = \ln\sqrt{3}$ or awrt 0.55	A1
	So, $x = -e^{-\frac{3}{2}\ln 3} + e^{-\frac{1}{2}\ln 3} = -e^{\ln 3^{-\frac{3}{2}}} + e^{\ln 3^{-\frac{1}{2}}}$ $x = -3^{-\frac{3}{2}} + 3^{-\frac{1}{2}}$	Substitutes their t back into x and an attempt to eliminate out the ln's.	ddM1
	$= -\dfrac{1}{3\sqrt{3}} + \dfrac{1}{\sqrt{3}} = \dfrac{2}{3\sqrt{3}} = \dfrac{2\sqrt{3}}{9}$	uses exact values to give $\dfrac{2\sqrt{3}}{9}$	A1 **AG**
	$\dfrac{d^2x}{dt^2} = -9e^{-3t} + e^{-t}$ At $t = \frac{1}{2}\ln 3$, $\dfrac{d^2x}{dt^2} = -9e^{-\frac{3}{2}\ln 3} + e^{-\frac{1}{2}\ln 3}$	Finds $\dfrac{d^2x}{dt^2}$ and substitutes their t into $\dfrac{d^2x}{dt^2}$	dM1*
	$= -9(3)^{-\frac{3}{2}} + 3^{-\frac{1}{2}} = -\dfrac{9}{3\sqrt{3}} + \dfrac{1}{\sqrt{3}} = -\dfrac{3}{\sqrt{3}} + \dfrac{1}{\sqrt{3}}$		
	As $\dfrac{d^2x}{dt^2} = -\dfrac{9}{3\sqrt{3}} + \dfrac{1}{\sqrt{3}} = \left\{-\dfrac{2}{\sqrt{3}}\right\} < 0$ then x is maximum.	$-\dfrac{9}{3\sqrt{3}} + \dfrac{1}{\sqrt{3}} < 0$ and maximum conclusion.	A1
			(7)
			[15]

June 2009
6669 Further Pure Mathematics FP3 (new)
Mark Scheme

Question Number	Scheme	Marks
Q1	$\frac{7}{\cosh x} - \frac{\sinh x}{\cosh x} = 5 \Rightarrow \frac{14}{e^x + e^{-x}} - \frac{(e^x - e^{-x})}{e^x + e^{-x}} = 5$	M1
	$\therefore 14 - (e^x - e^{-x}) = 5(e^x + e^{-x}) \Rightarrow 6e^x - 14 + 4e^{-x} = 0$	A1
	$\therefore 3e^{2x} - 7e^x + 2 = 0 \Rightarrow (3e^x - 1)(e^x - 2) = 0$	M1
	$\therefore e^x = \frac{1}{3}$ or 2	A1
	$x = \ln(\frac{1}{3})$ or $\ln 2$	B1ft **[5]**
Alternative (i)	Write $7 - \sinh x = 5\cosh x$, then use exponential substitution	M1
	$7 - \frac{1}{2}(e^x - e^{-x}) = \frac{5}{2}(e^x + e^{-x})$	
	Then proceed as method above.	
Alternative (ii)	$(7\operatorname{sech} x - 5)^2 = \tanh^2 x = 1 - \operatorname{sech}^2 x$	M1
	$50\operatorname{sech}^2 x - 70\operatorname{sech} x + 24 = 0$	A1
	$2(5\operatorname{sech} x - 3)(5\operatorname{sech} x - 4) = 0$	M1
	$\operatorname{sech} x = \frac{3}{5}$ or $\operatorname{sech} x = \frac{4}{5}$	A1
	$x = \ln(\frac{1}{3})$ or $\ln 2$	B1ft
Q2 (a)	$\mathbf{b} \times \mathbf{c} = 0\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$	M1 A1 A1 (3)
(b)	$\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = 0 + 5 = 5$	M1 A1 ft (2)
(c)	Area of triangle $OBC = \frac{1}{2}\lvert 5\mathbf{j} + 5\mathbf{k}\rvert = \frac{5}{2}\sqrt{2}$	M1 A1 (2)
(d)	Volume of tetrahedron $= \frac{1}{6} \times 5 = \frac{5}{6}$	B1 ft (1) **[8]**

Question Number	Scheme	Marks
Q3 (a)	$\begin{vmatrix} 6-\lambda & 1 & -1 \\ 0 & 7-\lambda & 0 \\ 3 & -1 & 2-\lambda \end{vmatrix} = 0 \quad \therefore (6-\lambda)(7-\lambda)(2-\lambda)+3(7-\lambda)=0$	M1
	$(7-\lambda)=0$ verifies $\lambda = 7$ is an eigenvalue (can be seen anywhere)	M1
	$\therefore (7-\lambda)\{12-8\lambda+\lambda^2+3\}=0 \quad \therefore (7-\lambda)\{\lambda^2-8\lambda+15\}=0$	A1
	$\therefore (7-\lambda)(\lambda-5)(\lambda-3)=0$ and 3 and 5 are the other two eigenvalues	M1 A1
		(5)
(b)	Set $\begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 7\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 0 \\ 3 & -1 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	M1
	Solve $-x+y-z=0$ and $3x-y-5z=0$ to obtain $x = 3z$ or $y = 4z$ and a second equation which can contain 3 variables	M1 A1
	Obtain eigenvector as $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ (or multiple)	A1
		(4)
		[9]

Question Number	Scheme	Marks
Q4 (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}} \times \frac{1}{\sqrt{1+(\sqrt{x})^2}}$	B1, M1
	$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1+x}} \quad \left(= \frac{1}{2\sqrt{x(1+x)}}\right)$	A1 (3)
(b)	$\therefore \int_{\frac{1}{4}}^{4} \frac{1}{\sqrt{x(x+1)}}\mathrm{d}x = \left[2\operatorname{ar}\sinh\sqrt{x}\right]_{\frac{1}{4}}^{4}$	M1
	$= \left[2\operatorname{ar}\sinh 2 - 2\operatorname{ar}\sinh(\frac{1}{2})\right]$	M1
	$= \left[2\ln(2+\sqrt{5})\right] - \left[2\ln(\frac{1}{2}+\sqrt{\frac{5}{4}})\right]$	M1
	$2\ln\frac{(2+\sqrt{5})}{(\frac{1}{2}+\sqrt{(\frac{5}{4})})} = 2\ln\frac{2(2+\sqrt{5})}{(1+\sqrt{5})} = 2\ln\frac{2(\sqrt{5}+2)(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)} = 2\ln\frac{(3+\sqrt{5})}{2}$	M1
	$= \ln\frac{(3+\sqrt{5})(3+\sqrt{5})}{4} = \ln\frac{14+6\sqrt{5}}{4} = \ln\left(\frac{7}{2}+\frac{3\sqrt{5}}{2}\right)$	A1 A1 (6) **[9]**
Alternative (i) for part (a)	Use $\sinh y = \sqrt{x}$ and state $\cosh y\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}}$	B1
	$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1+\sinh^2 y}} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{\left(1+\left(\sqrt{x}\right)^2\right)}}$	M1
	$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1+x}} \quad \left(= \frac{1}{2\sqrt{x(1+x)}}\right)$	A1 (3)
Alternative (i) for part (b)	Use $x = \tan^2\theta$, $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\tan\theta\sec^2\theta$ to give $2\int\sec\theta\,\mathrm{d}\theta = [2\ln(\sec\theta+\tan\theta]$	M1
	$= [2\ln(\sec\theta+\tan\theta]_{\tan\theta=\frac{1}{2}}^{\tan\theta=2}$ i.e. use of limits	M1
	then proceed as before from line 3 of scheme	
Alternative (ii) for part (b)	Use $\int\frac{1}{\sqrt{[(x+\frac{1}{2})^2-\frac{1}{4}]}}\mathrm{d}x = \operatorname{arcosh}\frac{x+\frac{1}{2}}{\frac{1}{2}}$	M1
	$= \left[\operatorname{arcosh}9 - \operatorname{arcosh}(\frac{3}{2})\right]$	M1
	$= \left[\ln(9+\sqrt{80})\right] - \left[\ln(\frac{3}{2}+\frac{1}{2}\sqrt{5})\right]$	M1
	$= \ln\frac{(9+\sqrt{80})}{(\frac{3}{2}+\frac{1}{2}\sqrt{5})} = \ln\frac{2(9+\sqrt{80})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$,	M1
	$= \ln\frac{2(9+4\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} = \ln\left(\frac{7}{2}+\frac{3\sqrt{5}}{2}\right)$	A1 A1 (6) **[9]**

Question Number	Scheme	Marks
Q5 (a)	$-(25-x^2)^{\frac{1}{2}}$ (+c)	M1A1 (2)
(b)	$I_n = \int x^{n-1}.\frac{x}{\sqrt{(25-x^2)}}\mathrm{d}x = -x^{n-1}\sqrt{25-x^2} + \int (n-1)x^{n-2}\sqrt{(25-x^2}\mathrm{d}x$	M1 A1ft
	$I_n = \left[-x^{n-1}\sqrt{25-x^2}\right]_0^5 + \int_0^5 \frac{(n-1)x^{n-2}(25-x^2)}{\sqrt{(25-x^2)}}\mathrm{d}x$	M1
	$I_n = 0 + 25(n\text{-}1)\ I_{n-2}\ \text{-}\ (n\text{-}1)\ I_n$	M1
	$\therefore nI_n = 25(n-1)I_{n-2}$ and so $I_n = \frac{25(n-1)}{n}I_{n-2}$ ✳	A1 (5)
(c)	$I_0 = \int_0^5 \frac{1}{\sqrt{(25-x^2}}\mathrm{d}x = \left[\arcsin(\tfrac{x}{5})\right]_0^5 = \frac{\pi}{2}$	M1 A1
	$I_4 = \frac{25\times 3}{4}\times\frac{25\times 1}{2}I_0 = \frac{1875}{16}\pi$	M1 A1 (4) **[11]**
Alternative for (b)	Using substitution $x = 5\sin\theta$	
	$I_n = 5^n\int_0^{\frac{\pi}{2}}\sin^n\theta\mathrm{d}\theta = \left[-5^n\sin^{n-1}\theta\cos\theta\right]_0^{\frac{\pi}{2}} + 5^n(n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta\cos^2\theta\mathrm{d}\theta$	M1A1
	$= \left[-5^n\sin^{n-1}\theta\cos\theta\right]_0^{\frac{\pi}{2}} + 5^n(n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta(1-\sin^2\theta)\mathrm{d}\theta$	M1
	$I_n = 0 + 25(n\text{-}1)\ I_{n-2}\ \text{-}\ (n\text{-}1)\ I_n$	M1
	$\therefore nI_n = 25(n-1)I_{n-2}$ and so $I_n = \frac{25(n-1)}{n}I_{n-2}$ ✳	A1
	(need to see that $I_{n-2} = 5^{n-2}\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta\mathrm{d}\theta$ for final A1)	(5)

Question Number	Scheme	Marks
Q6 (a)	$\dfrac{x^2}{a^2} - \dfrac{(mx+c)^2}{b^2} = 1$ and so $b^2x^2 - a^2(mx+c)^2 = a^2b^2$	M1
	$\therefore\ (b^2 - a^2m^2)x^2 - 2a^2mcx - a^2(c^2+b^2) = 0$ Or $(a^2m^2 - b^2)x^2 + 2a^2mcx + a^2(c^2+b^2) = 0$ $\ast$	A1 (2)
(b)	$(2a^2mc)^2 = 4(a^2m^2 - b^2)\times a^2(c^2+b^2)$	M1
	$4a^4m^2c^2 = -4a^2(b^2c^2 + b^4 - a^2m^2c^2 - a^2m^2b^2)$	
	$c^2 = a^2m^2 - b^2$ or $a^2m^2 = b^2 + c^2$ $\ast$	A1 (2)
(c)	Substitute (1, 4) into $y = mx+c$ to give $4 = m + c$ and	
	Substitute $a = 5$ and $b = 4$ into $c^2 = a^2m^2 - b^2$ to give $c^2 = 25m^2 - 16$	B1
	Solve simultaneous equations to eliminate m or c : $(4-m)^2 = 25m^2 - 16$	M1
	To obtain $24m^2 + 8m - 32 = 0$	A1
	Solve to obtain $8(3m+4)(m-1) = 0$......$m =$ ..or...	M1
	$m = 1$ or $-\dfrac{4}{3}$	A1
	Substitute to get $c = 3$ or $\dfrac{16}{3}$	M1
	Lines are $y = x + 3$ and $3y + 4x = 16$	A1 (7) **[11]**

Question Number	Scheme	Marks
Q7 (a)	If the lines meet, $-1+3\lambda=-4+3\mu$ and $2+4\lambda=2\mu$	M1
	Solve to give $\lambda=0$ $(\mu=1$ but this need not be seen$)$.	M1 A1
	Also $1-\lambda=\alpha$ and so $\alpha=1$.	B1 (4)
(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 4 \\ 0 & 3 & 2 \end{vmatrix}$ = -6**i**+2**j**-3**k** is perpendicular to both lines and hence to the plane	M1 A1
	The plane has equation **r.n**=**a.n** , which is $-6x+2y-3z=-14$,	M1
	i.e. $-6x+2y-3z+14=0$.	A1 o.a.e. (4)
OR (b)	**Alternative scheme** Use (1, -1,2) and (α ,-4,0) in equation $ax+by+cz+d=0$ And third point so three equations, and attempt to solve Obtain $6x-2y+3z=$ $(6x-2y+3z)-14=0$	M1 M1 A1 A1 o.a.e. (4)
(c)	$(\mathbf{a}_1-\mathbf{a}_2)=\mathbf{i}-3\mathbf{j}-2\mathbf{k}$	M1
	Use formula $\dfrac{(\mathbf{a}_1-\mathbf{a}_2)\bullet\mathbf{n}}{\lvert\mathbf{n}\rvert}=\dfrac{(\mathbf{i}-3\mathbf{j}-2\mathbf{k})\bullet(-6\mathbf{i}+2\mathbf{j}-3\mathbf{k})}{\sqrt{(36+4+9)}}=\left(\dfrac{-6}{7}\right)$	M1
	Distance is $\dfrac{6}{7}$	A1 (3) **[11]**

Question Number	Scheme	Marks
Q8 (a)	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -3\sin\theta,\ \frac{\mathrm{d}y}{\mathrm{d}\theta} = 5\cos\theta$	B1
	so $S = 2\pi\int 5\sin\theta\sqrt{(-3\sin\theta)^2 + (5\cos\theta)^2}\,\mathrm{d}\theta$	M1
	$\therefore S = \quad 2\pi\int 5\sin\theta\sqrt{9 - 9\cos^2\theta + 25\cos^2\theta}\,\mathrm{d}\theta$	M1
	Let $c = \cos\theta,\ \frac{dc}{d\theta} = -\sin\theta$, limits 0 and $\frac{\pi}{2}$ become 1 and 0	M1
	So $S = k\pi\int_0^{\alpha}\sqrt{16c^2 + 9}\,\mathrm{d}c$, where $k = 10$, and α is 1	A1, A1 (6)
(b)	Let $c = \frac{3}{4}\sinh u$. Then $\frac{dc}{du} = \frac{3}{4}\cosh u$	M1
	So $S = k\pi\int_?^?\sqrt{9\sinh^2 u + 9}\ \frac{3}{4}\cosh u\mathrm{d}u$	A1
	$= k\pi\int_?^?\ \frac{9}{4}\cosh^2 u\,\mathrm{d}u = k\pi\int_?^?\ \frac{9}{8}(\cosh 2u + 1)\mathrm{d}u$	M1
	$= k\pi\left[\frac{9}{16}\sinh 2u + \frac{9}{8}u\right]_0^d$	A1
	$= \frac{45\pi}{4}\left[\frac{20}{9} + \ln 3\right]$ i.e. $\underline{117}$	B1
		(5)
		[11]

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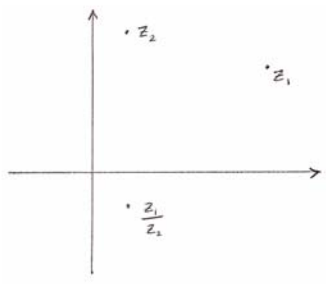
6674 Further Pure Mathematics FP1 (legacy)
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	$x = -\mathrm{i}$ is a root (Scored here or in (b))	1B1
	Factor $(x+\mathrm{i})(x-\mathrm{i}) = x^2+1$	2B1
	$x^4+6x^3+26x^2+6x+25 = (x^2+1)(x^2+6x+25)$	1M1 1A1 (4)
(b)	Solving quadratic: $x = \dfrac{-6 \pm \sqrt{36-100}}{2}$	1M1
	$x = -3 \pm 4\mathrm{i}$	1A1 (2) **[6]**
(a)	1B1 CAO, x = -i, maybe seen in (b) 2B1 x^2+1 CAO 1M1 Getting the three term quadratic 1A1 CAO for correct quadratic	
(b)	1M1 Solving a three term quadratic to x = complex, correct formula used 1A1 CAO	

Question Number	Scheme	Marks
Q2	$m^2+6m+10=0 \qquad m=\dfrac{-6\pm\sqrt{36-40}}{2}=-3\pm \mathrm{i}$	1B1
	C.F. $(x=)\ \mathrm{e}^{-3t}(A\cos t+B\sin t)$	1M1 1A1ft
	P.I. $x=k\mathrm{e}^{-4t}$	2B1
	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=-4k\mathrm{e}^{-4t} \qquad \dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}=16k\mathrm{e}^{-4t}$	2M1
	$16k-24k+10k=1 \qquad k=\dfrac{1}{2}$	3M1 2A1
	General solution: $x=\mathrm{e}^{-3t}(A\cos t+B\sin t)+\dfrac{1}{2}\mathrm{e}^{-4t}$	3A1ft=3B1ft
		[8]
	1B1 CAO (may be implied) 1M1 Correct 'shape' $e^{at}(A\cos bt+B\sin bt)$ accept alterative (single) variable here. No complex 1A1ft condone their variables 2B1 CAO 2M1 Attempt at both, accept ke^{-at} (a>0) derivatives here. 3M1 Linear in k, to k = 2A1 CAO 3A1ft = 3B1ft but must be x = f(t).	

Question Number	Scheme	Marks
Q3 (a)	$r(r+2)(r+4) = r^3 + 6r^2 + 8r$, so use $\sum r^3 + 6\sum r^2 + 8\sum r$	1M1
	$= \frac{1}{4}n^2(n+1)^2 + 6\left(\frac{1}{6}n(n+1)(2n+1)\right) + 8\left(\frac{1}{2}n(n+1)\right)$	1A1
	$= \frac{1}{4}n(n+1)\{n(n+1) + 4(2n+1) + 16\}$	2M1 2A1
	$= \frac{1}{4}n(n+1)\{n^2 + 9n + 20\} = \frac{1}{4}n(n+1)(n+4)(n+5)$ (*)	3A1 (5)
(b)	$\sum_{21}^{30} = \sum_{1}^{30} - \sum_{1}^{20}$	1M1
	$= \frac{1}{4}(30\times 31\times 34\times 35) - \frac{1}{4}(20\times 21\times 24\times 25) = 213675$	1A1 (2)
		[7]
(a)	Alternative (induction): $\frac{1}{4}k(k+1)(k+4)(k+5) + (k+1)(k+3)(k+5)$ 1M1 (Adding on (k+1)th term $= \frac{1}{4}(k+1)(k+5)(k^2 + 4k + 4k + 12)$ 2M1 Quadratic factor seen $= \frac{1}{4}(k+1)(k+2)(k+5)(k+6)$ 1A1 cso Check for k = 1: Term = 15, Sum = $\frac{60}{4} = 15$ 1B1 cao Induction argument + conclusion 2A1 cao Q3 Notes 1M1 Expand in terms of $\sum r^3, \sum r^2, \sum r$ 1A1 Correct substitution in correct expansion. 2M1 Factorisation, 3 term quadratic factor seen 2A1 a correct quadratic factor 3A1 cso	
(b)	1M1 allowed for $\sum_{21}^{30} = \sum_{1}^{30} - \sum_{1}^{19}$ or $\sum_{21}^{30} = \sum_{1}^{30} - \sum_{1}^{21}$ but must be used. 1A1 cao	

Question Number	Scheme	Marks
Q4 (a)	$z_2 = \dfrac{z_1}{1-\mathrm{i}} = \dfrac{5+2p\mathrm{i}}{1-\mathrm{i}} \times \dfrac{1+\mathrm{i}}{1+\mathrm{i}}$	1 M1
	$\dfrac{(5-2p)+\mathrm{i}(5+2p)}{2} = \left(\dfrac{5-2p}{2}\right), +\mathrm{i}\left(\dfrac{5+2p}{2}\right)$	1A1,2A1 (3)
(b)	$\dfrac{5+2p}{5-2p} = 4$ $\quad 5+2p = 20-8p \quad p = \dfrac{3}{2}$	1M1 1A1ft (2)
(c)	$\lvert z_2 \rvert = \sqrt{1^2+4^2} = \sqrt{17} = 4.12$	1M1 1A1 (2)
(d)		
	For $\dfrac{z_1}{z_2}$	1B1
	For z_1 and z_2 $(z_1 = 5+3\mathrm{i}$ and $z_2 = 1+4\mathrm{i})$	2B1ft (2) **[9]**
(a)	Alternative: $5+2p\mathrm{i} = (1-\mathrm{i})(a+b\mathrm{i})$ and equate real and imaginary parts M1 $(a+b=5$ and $b-a=2p)$	
(c)	Alternative: $\lvert z_2 \rvert = \dfrac{\lvert z_1 \rvert}{\sqrt{2}} = \dfrac{\sqrt{25+(2p)^2}}{\sqrt{2}}$ and substitute value for p. M1 Q4 Notes (a) 1M1 A correct method leading to coordinate 1A1 cao 2A1 cao (b) 1M1 linear equation in p, their Im/Re = 4 1A1ft from their (a) (c) 1M1 Pythagoras 1A1 cao (awrt 4.12) (d) 1B1 cao 2B1ft If points unlabelled withhold this mark, relative positions plausible	

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>Q5 (a)

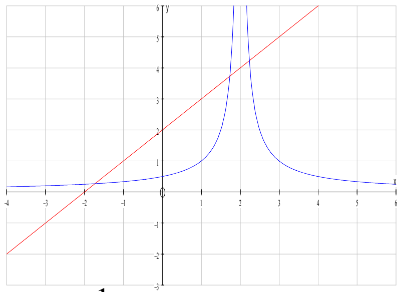
(b)

(c)</td><td>$f(0.8) = \sin 1.6 - \ln 2.4 \quad (= 0.1241...)$
$f(0.9) = \sin 1.8 - \ln 2.7 \quad (= -0.0194...)$
Values correct (to 1 s.f.). Change of sign $\Rightarrow$ Root
$f'(x) = 2\cos 2x, \ -\dfrac{1}{x}$
$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 0.9 - \dfrac{-0.0194}{-1.5655}, \quad = 0.888$

$\dfrac{0.1241}{k} = \dfrac{0.0194}{0.1-k}$ (where root is approx. 0.8 + k)
$k = 0.086...$ $\quad \alpha \approx 0.886$ (Allow awrt)</td><td>
1M1
1A1 (2)
1B1, 2B1
1M1 1A1 2A1
(5)
M1 1A1ft
2A1 (3)
[10]</td></tr>
<tr><td></td><td>Alternative for (c)
$\dfrac{0.9-\alpha}{0.0194} = \dfrac{\alpha - 0.8}{0.1241}$ M1 A1
$0.11169 - 0.1241\alpha = 0.0194\alpha - 0.01552$
$0.12721 = 0.1435\alpha$
$\alpha \approx 0.886$ A1

Q5 Notes
(a) 1M1 Both evaluated
1A1 cao including conclusion statement
(b) 1B1 2 cos2x cao
2B1 -1/x cao
1M1 Substituting values
1A1 cao 3 dp rounded or truncated
2A1 cao 0.888 gets both A marks
(c) 1M1 Accept sign errors here, accept f(0.8) and f(0.9)
1A1ft their values from (a), signs correct.
2A1 cao 0.886 gets both A marks</td><td></td></tr>
</table>

Question Number	Scheme	Marks
Q6 (a)	Integrating factor $e^{\int \cot x \, dx} = e^{\ln(\sin x)} = \sin x$	1M1
	$y\sin x = \int \sin^2 x \, dx$ or $\frac{d}{dx}(y\sin x) = \sin^2 x$	2M1 1A1
	$\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} dx = \frac{x}{2} - \frac{\sin 2x}{4} \quad (+C)$	3M1 2A1
	$y = \frac{2x - \sin 2x + C}{4\sin x}$ (or equiv.)	3A1 (6)
(b)	y = 1 at $x = \frac{\pi}{2}$: $1 = \frac{\pi + C}{4}$ $C = 4 - \pi$	1M1 1A1
	At $x = \frac{\pi}{4}$, $y = \frac{\frac{\pi}{2} - 1 + 4 - \pi}{\frac{4}{\sqrt{2}}} = \frac{\sqrt{2}}{4}\left(3 - \frac{\pi}{2}\right) = \frac{(6-\pi)\sqrt{2}}{8}$ (*)	2M1 2A1 (4)
		[10]
(a)	Alternative (special case):	
	Multiply by sin x and integrate 'by inspection' M2	
	Achieve $y\sin x = \int \sin^2 x \, dx$ or $\frac{d}{dx}(y\sin x) = \sin^2 x$ A1	
(b)	Note that other C values are possible,	
	e.g. from $y = \frac{2x - \sin 2x}{4\sin x} + \frac{C}{\sin x}$	
	Q6 Notes	
	(a) 1M1 Integrating factor found, condone sign error	
	2M1 One side correct	
	1A1 cao both sides correct	
	3M1 'RHS' in a form that can be integrated	
	2A1 'RHS' integrated cao	
	3A1 cao to y = , general solution	
	(b) 1M1 Substitute to find their C	
	1A1 their C cao	
	2M1 substitute to find y	
	2A1 cso	

Question Number	Scheme	Marks
Q7 (a)	 Line, positive grad., intercepts (0, 2), (-2, 0)	1B1
	Curve, branch x > 2	2B1
	Curve, branch x < 2	3B1
	Curve intercept $\left(0, \frac{1}{2}\right)$	4B1
	Asymptotes x = 2 and y = 0	1M1 1A1(6)
(b)	$x+2=\dfrac{1}{x-2}$ $\quad x^2-4=1$ $\quad x=\sqrt{5}$	1M1 1A1
	$x+2=\dfrac{1}{2-x}$ $\quad 4-x^2=1$ $\quad x=\sqrt{3}$	2M1 2A1
	$x<-\sqrt{3}$, $\quad \sqrt{3}<x<\sqrt{5}$	1B1ft, 2B1ft
		(6) **[12]**
	Special case (a) for $y=\left\lvert\dfrac{1}{x+2}\right\rvert$ allow 2B1 if both branches correct Q7 Notes (a) 1B1 cao intercepts clear 2B1 cao 3B1 cao 4B1 cao 1/2 indicated 1M1 One stated 1A1 both stated (b) 1M1 condone inequality here, seeking one critical value 1A1 finding 1st critical value, exact, but ignore signs 2M1 condone inequality here, seeking second critical value 2A1 finding 2nd critical value, exact, but ignore signs 1B1ft ft their values penalise $\leq$ once only at first occurrence 2B1ft ft their values condone $x \neq 2$.	

Question Number	Scheme	Marks
Q8 (a)	$r\sin\theta = \sin\theta + \sin\theta\cos\theta$	
	$\dfrac{\mathrm{d}(r\sin\theta)}{\mathrm{d}\theta} = \cos\theta + \cos 2\theta = \cos\theta + \cos^2\theta - \sin^2\theta$	1M1 1A1
	$2\cos^2\theta + \cos\theta - 1 = 0 \Rightarrow \cos\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3} \quad r = \dfrac{3}{2}$ (*)	2M1 2A1 (4)
(b)	$\dfrac{1}{2}\int r^2\,\mathrm{d}\theta = \dfrac{1}{2}\int\left(1 + 2\cos\theta + \cos^2\theta\right)\mathrm{d}\theta$	1 M1
	$\int\left(1 + 2\cos\theta + \cos^2\theta\right)\mathrm{d}\theta = \left[\theta + 2\sin\theta + \dfrac{\sin 2\theta}{4} + \dfrac{\theta}{2}\right]$	2M1 1A1
	$\left[\dfrac{3\theta}{2} + 2\sin\theta + \dfrac{\sin 2\theta}{4}\right]_0^{\pi/3} = \dfrac{\pi}{2} + \sqrt{3} + \dfrac{\sqrt{3}}{8} \quad \left(= \dfrac{\pi}{2} + \dfrac{9\sqrt{3}}{8}\right)$	3M1
	$AH = r\sin\theta = \dfrac{3}{2}\times\dfrac{\sqrt{3}}{2} = \dfrac{3\sqrt{3}}{4}$, $PH = 2 - r\cos\theta = 2 - \dfrac{3}{2}\times\dfrac{1}{2} = \dfrac{5}{4}$	1B1, 2B1
	Area of trapezium OAHP: $\dfrac{1}{2}\left(2 + \dfrac{5}{4}\right)\dfrac{3\sqrt{3}}{4} \quad \left(= \dfrac{39\sqrt{3}}{32}\right)$	4M1
	Area of R: $\dfrac{39\sqrt{3}}{32} - \left(\dfrac{\pi}{4} + \dfrac{9\sqrt{3}}{16}\right) = \dfrac{21\sqrt{3}}{32} - \dfrac{\pi}{4}$	5M1 2A1 (9)
		[13]
	Q8 Notes (a) 1M1 Finding rsinθ 1A1 cao 2M1 putting $\dfrac{\mathrm{d}(r\sin\theta)}{\mathrm{d}\theta} = 0$ to θ =, accept substitution of θ to show derivative =0 2A1 cso (b) 1M1 $\frac{1}{2}\int r^2 d\theta$ in terms of θ, expanded. 2M1 integrating, at least 1 trig term correctly handled 1A1 cao 3M1 substituting correct limits 1B1 $3\sqrt{3}/4$ cao careful, may be on diagram 2B1 5/4 or ¾ cao careful, may be on diagram 4M1 Trapezium or $\left(\frac{1}{2}\times\frac{3}{4}\times\frac{3\sqrt{3}}{4}\right) + \left(\frac{5}{4}\times\frac{3\sqrt{3}}{4}\right) = \frac{9\sqrt{3}}{32} + \frac{15\sqrt{3}}{32} = \frac{39\sqrt{3}}{32}$ 5M1 Subtracting their integral and their trapezium 2A1 cao	

June 2009
6675 Further Pure Mathematics FP2 (legacy)
Mark Scheme

Question Number	Scheme	Marks
Q1	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2 \times \operatorname{arsinh} 2x \times \frac{2}{\sqrt{(4x^2+1)}}$	M1 A1
	At $x = \frac{1}{2}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{\sqrt{2}} \operatorname{arsinh} 1$	M1 A1ft
	$= 2\sqrt{2} \ln\left(\sqrt{2}+1\right)$	A1 (5)
		[5]
	Alternative	
	$\sinh y^{\frac{1}{2}} = 2x$	
	$\frac{1}{2} y^{-\frac{1}{2}} \cosh y^{\frac{1}{2}} \frac{\mathrm{d}y}{\mathrm{d}x} = 2$	M1 A1
	$\sqrt{\left(1+\sinh^2 y^{\frac{1}{2}}\right)} \frac{\mathrm{d}y}{\mathrm{d}x} = 4y^{\frac{1}{2}}$	
	At $x = \frac{1}{2}$, $\sinh y^{\frac{1}{2}} = 1$	
	$\sqrt{(1+1)} \frac{\mathrm{d}y}{\mathrm{d}x} = 4 \operatorname{arsinh} 1$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{\sqrt{2}} \operatorname{arsinh} 1$	A1ft
	$= 2\sqrt{2} \ln\left(\sqrt{2}+1\right)$	A1 (5)
Q2 (a)	$b^2 = a^2\left(1-e^2\right) \Rightarrow 8 = a^2\left(1-\frac{1}{2}\right) \Rightarrow a = 4$	M1 A1 (2)
(b)	At S, $x = ae = 2\sqrt{2}$; at D, $y = 2\sqrt{2}$ two coordinates	B1
	($SDS'D'$ is a square)	
	$A = 4 \times \frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2} = 16$	M1 A1 (3)
		[5]

Question Number	Scheme	Marks
Q3 (a)	$\int_0^1 (1-x)^n \cosh x \, dx = \left[(1-x)^n \sinh x\right]_0^1 + \int_0^1 n(1-x)^{n-1} \sinh x \, dx$	M1
	$= \int_0^1 n(1-x)^{n-1} \sinh x \, dx$	A1
	$= \left[n(1-x)^{n-1} \cosh x\right]_0^1 + \int_0^1 n(n-1)(1-x)^{n-2} \cosh x \, dx$	M1
	$= -n + n(n-1)\int_0^1 (1-x)^{n-2} \cosh x \, dx$	M1
	$I_n = n(n-1)I_{n-2} - n$ ✱ cso	A1 (5)
(b)	$I_0 = \int_0^1 \cosh x \, dx = [\sinh x]_0^1 = \sinh 1 \quad \left(= \frac{1}{2}(e - e^{-1})\right)$	B1
	$I_2 = 2I_0 - 2$	M1
	$I_4 = 12I_2 - 4 = 24I_0 - 28$	M1
	$= 12e - \frac{12}{e} - 28$	A1 (4)
		[9]
Q4 (a)	$\frac{dy}{dx} = 15\cosh x - 17\sinh x + 6$	B1
	$\frac{dy}{dx} = 0 \Rightarrow 15\left(\frac{e^x + e^{-x}}{2}\right) - 17\left(\frac{e^x - e^{-x}}{2}\right) + 6 = 0$	M1
	$e^{2x} - 6e^x - 16 = 0$	M1 A1
	$(e^x - 8)(e^x + 2) = 0$	M1
	$x = 3\ln 2$	A1 (6)
(b)	$\frac{d^2y}{dx^2} = 15\sinh x - 17\cosh x$	M1
	$= -e^x - 16e^{-x} < 0$ (for any real x)	M1
	$\Rightarrow$ maximum	A1 (3)
	Accept equivalent arguments or a sketch	
		[9]

Question Number	Scheme	Marks	
Q5	Use of $S = 2\pi \int y\left(\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2\right)^{\frac{1}{2}} \mathrm{d}t$	B1	
	$\int y\sqrt{\left(\dot{x}^2+\dot{y}^2\right)}\mathrm{d}t = \int 3t^2\sqrt{\left(36t^4+36t^2\right)}\mathrm{d}t$	M1 A1	
	$= \int 18t^3\sqrt{\left(t^2+1\right)}\mathrm{d}t$		
	Let $u^2 = t^2+1$, $u\frac{\mathrm{d}u}{\mathrm{d}t} = t$		
	$\int t^3\sqrt{\left(t^2+1\right)}\mathrm{d}t = \int\left(u^2-1\right)u^2\,\mathrm{d}u$	M1 A1	
	$= \left(\frac{u^5}{5} - \frac{u^3}{3}\right)$	M1 A1	
	$\left[\left(\frac{u^5}{5} - \frac{u^3}{3}\right)\right]_1^{\sqrt{2}} = \frac{1}{15}\left(2\sqrt{2}-(-2)\right)$ using correct limits	M1	
	Leading to $A = \frac{24\pi}{5}\left(\sqrt{2}+1\right)$ ✱ cso	A1	(9)
			[9]
	Alternative substitutions		
	① Let $u = t^2+1$, $\frac{\mathrm{d}u}{\mathrm{d}t} = 2t$		
	$\int t^3\sqrt{\left(t^2+1\right)}\mathrm{d}t = \frac{1}{2}\int(u-1)u^{\frac{1}{2}}\,\mathrm{d}u$	M1 A1	
	$= \frac{1}{2}\int\left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u = \frac{1}{2}\left(\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right)$	M1 A1	
	Using the limits $u=1$ and $u=2$	M1	
	Leading to $A = \frac{24\pi}{5}\left(\sqrt{2}+1\right)$ ✱ cso	A1	
	② Let $t = \sinh u$, $\frac{\mathrm{d}t}{\mathrm{d}u} = \sinh u$		
	$\int t^3\sqrt{\left(t^2+1\right)}\mathrm{d}t = \int \sinh^3 u\cosh^2 u\,\mathrm{d}u$	M1 A1	
	$= \int\left(\cosh^4 u - \cosh^2 u\right)\sinh u\,\mathrm{d}u = \frac{\cosh^5 u}{5} - \frac{\cosh^3 u}{3}$	M1 A1	
	Using the limits $\cosh u = 1$ and $\cosh u = \sqrt{2}$	M1	
	Leading to $A = \frac{24\pi}{5}\left(\sqrt{2}+1\right)$ ✱ cso	A1	

Question Number	Scheme	Marks
Q6	$u = \cosh\theta \quad \Rightarrow \quad \dfrac{du}{d\theta} = \sinh\theta$	B1
	$I = \displaystyle\int \frac{u+1}{\sinh^2\theta(u-1)^2}\,du$	M1
	$= \displaystyle\int \frac{u+1}{(u^2-1)(u-1)^2}\,du$	M1
	$= \displaystyle\int \frac{1}{(u-1)^3}\,du$	A1
	$= -\dfrac{1}{2(u-1)^2}$	M1 A1
	At $\theta = \ln 4$, $u = \dfrac{4+\frac{1}{4}}{2} = \dfrac{17}{8}$; at $\theta = \ln 2$, $u = \dfrac{2+\frac{1}{2}}{2} = \dfrac{5}{4}$ both	M1 A1
	$\left[-\dfrac{1}{2(u-1)^2}\right]_{5/4}^{17/8} = 8 - \dfrac{32}{81} = \dfrac{616}{81}$	M1 A1 (10)
		[10]

Question Number	Scheme	Marks
Q7 (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos x}{\sin x} \left(= \cot x\right)$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \tan\psi = \cot x$	M1
	$\tan\psi = \tan\left(\frac{\pi}{2} - x\right) \Rightarrow \psi = \frac{\pi}{2} - x$ ✱ cso	A1 (3)
(b)	$s = \int\left(1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)^{\frac{1}{2}} \mathrm{d}x = \int\left(1 + \cot^2 x\right)^{\frac{1}{2}} \mathrm{d}x$	M1
	$= \int \operatorname{cosec} x \,\mathrm{d}x$	A1
	$= -\ln\left(\operatorname{cosec} x + \cot x\right)\left(+C\right)$	A1
	$= -\ln\left(\sec\psi + \tan\psi\right)\left(+C\right)$	M1
	$\left(0, \frac{\pi}{4}\right) \Rightarrow 0 = -\ln\left(\sqrt{2} + 1\right) + C$	M1
	$s = \ln\left(\frac{\sqrt{2}+1}{\sec\psi + \tan\psi}\right)$ ✱ cso	A1 (6)
(c)	$\frac{\mathrm{d}s}{\mathrm{d}\psi} = -\sec\psi$	M1
	$\psi = \frac{\pi}{6} \Rightarrow \rho = \left\vert\frac{\mathrm{d}s}{\mathrm{d}\psi}\right\vert = \frac{2}{\sqrt{3}}$ awrt 1.15	M1 A1 (3)
		[12]
	Alternative to (c)	
	$\psi = \frac{\pi}{6} \Rightarrow x = \frac{\pi}{3}$	
	At $x = \frac{\pi}{3}$; $\frac{\mathrm{d}y}{\mathrm{d}x} = \cot x = \frac{1}{\sqrt{3}}$, $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\operatorname{cosec}^2 x = -\frac{4}{3}$ both	M1
	$\rho = \left\vert\frac{\left(1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)^{\frac{3}{2}}}{\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}}\right\vert = \frac{\left(1 + \frac{1}{3}\right)^{\frac{3}{2}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}}$ awrt 1.15	M1 A1 (3)

Question Number	Scheme		Marks	
Q8 (a)	$\frac{\mathrm{d}x}{\mathrm{d}p}=2ap,\ \frac{\mathrm{d}y}{\mathrm{d}p}=2a;\quad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{p}$		M1 A1	
	$y-2ap=-p\left(x-ap^2\right)$		M1	
	$y+px=2ap+ap^3$ ✱	cso	A1	(4)
(b)	Eliminating x between $y^2=4ax$ and $y+px=2ap+ap^3$			
	$y+\frac{py^2}{4a}=2ap+ap^3$		M1	
	$py^2+4ay-8a^2p-4a^2p^3=0$		A1	
	$\left(y-2ap\right)\left(py+4a+2ap^2\right)=0$		M1 A1	
	At Q, $y=-\frac{4a+2ap^2}{p}=-2a\left(\frac{2+p^2}{p}\right)$ ✱	cso	A1	(5)
(c)	At Q, $x=a\left(\frac{2+p^2}{p}\right)^2$			
	$PQ^2=\left(ap^2-a\left(\frac{2+p^2}{p}\right)^2\right)^2+\left(2ap+2a\left(\frac{2+p^2}{p}\right)\right)^2$		M1 A1	
	$=\frac{16a^2\left(p^2+1\right)^3}{p^4}$			
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(PQ^2\right)=16a^2\left(\frac{6\left(p^2+1\right)^2p^5-\left(p^2+1\right)^3.4p^3}{p^8}\right)$		M1	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(PQ^2\right)=0\ \Rightarrow\ \frac{2\left(p^2+1\right)^2\left(p^2-2\right)}{p^5}=0$			
	$p=(\pm)\sqrt{2}$		M1 A1	
	$PQ^2=\frac{16a^2\times 27}{4}$		M1	
	$PQ_{\min}=6\sqrt{3}a$ ✱	cso	A1	(7)
				[16]

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June 2009
6676 Further Pure Mathematics FP3 (legacy)
Mark Scheme

Question Number	Scheme	Marks
Q1	At $x = 0.1$, $y_1 = 0.1(0 \text{ x } 0 + 3) + 0 = 0.3$	B1
	$x = 0.2$, $y_2 = 0.1\ (\ 0.1 \text{ x } 0.3^2 + 3) + 0.3$	M1
	$(= 0.3009 + 0.3)$	
	$= 0.6009$	A1
	$x = 0.3$, $y_3 = 0.1\ (\ 0.2 \text{ x } 0.6009^2 + 3) + 0.6009$	M1
	$(= 0.307221616........+ 0.6009)$	
	$= 0.908(121616.......)$ Allow awrt 0.908	A1
		[5]
Q2 (a)	$\mathbf{b} \times \mathbf{c} = 0\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$	M1 A1 A1
		(3)
(b)	$\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} . \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix} = 0 + 5 + 0 = 5$	M1 A1ft
		(2)
(c)	Area of triangle $OBC = \frac{1}{2}\lvert 5\mathbf{j} + 5\mathbf{k} \rvert = \frac{5}{2}\sqrt{2}$ oe	M1 A1
		(2)
(d)	Volume of tetrahedron $= \frac{1}{6} \times 5 = \frac{5}{6}$	B1 ft
		(1)
		[8]

Question Number	Scheme	Marks
Q3 (a)	$\begin{vmatrix} 6-\lambda & 1 & -1 \\ 0 & 7-\lambda & 0 \\ 3 & -1 & 2-\lambda \end{vmatrix} = 0$	
	$\therefore (6-\lambda)((7-\lambda)(2-\lambda)-0)-1\times 0-1(0-3(7-\lambda))=0$ $\therefore (6-\lambda)(7-\lambda)(2-\lambda)+3(7-\lambda)=0$	M1
	$(7-\lambda)=0$ verifies $\lambda=7$ is an eigenvalue	M1
	They may show $\lambda = 7$ in the determinant (e.g. -1(0 – 0)-1(0 – 0)-1(0 – 0)) $\therefore (7-\lambda)\left\{12-8\lambda+\lambda^2+3\right\}=0$	A1
	$\therefore (7-\lambda)\left\{\lambda^2-8\lambda+15\right\}=0$ (NB $\therefore \lambda^3-15\lambda^2+71\lambda-105=0$)	
	$\therefore (7-\lambda)(\lambda-5)(\lambda-3)=0$ and 3 and 5 are the other two eigenvalues	M1 A1
		(5)
(b)	$\begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 7\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 0 \\ 3 & -1 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	M1
	$-x+y-z=0$ $(0=0)$ $3x-y-5z=0$ Solves to obtain $x = 3z$ and $y = 4z$ (3y = 4x) or equivalent	M1 A1
	Obtains eigenvector as $\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ (or multiple)	A1
		(4) **[9]**

Question Number	Scheme	Marks
Q4 (a)	$\frac{d^2y}{dx^2}+2\times 2+1=1$, and so $\frac{d^2y}{dx^2}=-4$ at $x=0$.	B1
	$y'''+\{(1+y^2)y''+2y(y')(y')\}+y'=2e^{2x}$	M1 {M1 A1} A1
	$y'''+(1+1)(-4)+2\times 1(2)(2)+2=2$, i.e. $y'''=0$	B1 cso
		(6)
(b)	$y=1,+2x(+...)$	B1,B1
	$-\frac{4x^2}{2}+\frac{0x^3}{6}+\frac{40x^4}{24}$	M1 A1
	$(=-2x^2+\frac{5x^4}{3})$	
		(4)
		[10]
Q5 (a)		
	$\cos 6\theta=\text{Re}[(\cos\theta+i\sin\theta)^6]$	M1
	$(\cos\theta+\text{i}\sin\theta)^6=c^6+6c^5\text{i}s+15c^4\text{i}^2s^2+20c^3\text{i}^3s^3+15c^2\text{i}^4s^4+6c\text{i}^5s^5+\text{i}^6s^6$	
	$\cos 6\theta=c^6-15c^4s^2+15c^2s^4-s^6$	M1 A1
	$=c^6-15c^4(1-c^2)+15c^2(1-c^2)^2-(1-c^2)^3$	M1
	$\cos 6\theta=c^6-15c^4+15c^6+15c^2(1-2c^2+c^4)\ -(1-3c^2+3c^4-c^6)$	
	$\cos 6\theta=32\cos^6\theta-48\cos^4\theta+18\cos^2\theta-1$ *	A1
		(5)
(b)		
	$\cos 6\theta=\cos 2\theta\ \rightarrow 32\cos^6\theta-48\cos^4\theta+18\cos^2\theta-1=2\cos^2\theta-1$	M1 A1
	$32\cos^6\theta-48\cos^4\theta+16\cos^2\theta=0$	
	$16\cos^2\theta(2\cos^4\theta-3\cos^2\theta+1)=0$	M1
	$(2\cos^2\theta-1)(\cos^2\theta-1)=0$	
	$\therefore\cos^2\theta=0,\ \frac{1}{2}$ or 1 so $\cos\theta=0,\ \pm\frac{1}{\sqrt{2}}$ or ± 1	M1
	Uses arccos on at least 3 different values	
	$\therefore\theta=0,\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4}$ and π	A1,A1
	Decimals: Allow 0, 0.785, 1.57, 2.36, 3.14 (awrt)	
	3correct solutions A1, all correct A1	
		(6)
		[11]

Question Number	Scheme	Marks
Q6 (a)	When $n = 1$ LHS = RHS = $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. Result true for $n = 1$	B1
	Assume result true for $n = k$ i.e. $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^k = \begin{pmatrix} \cos k\theta & -\sin k\theta \\ \sin k\theta & \cos k\theta \end{pmatrix}$	
	And multiply both sides by $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$	
	Then $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^{k+1} = \begin{pmatrix} \cos k\theta & -\sin k\theta \\ \sin k\theta & \cos k\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$	M1
	$= \begin{pmatrix} \cos k\theta\cos\theta - \sin k\theta\sin\theta & -\cos k\theta\sin\theta - \sin k\theta\cos\theta \\ \sin k\theta\cos\theta + \cos k\theta\sin\theta & -\sin k\theta\sin\theta + \cos k\theta\cos\theta \end{pmatrix}$	M1
	i.e. $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^{k+1} = \begin{pmatrix} \cos(k+1)\theta & -\sin(k+1)\theta \\ \sin(k+1)\theta & \cos(k+1)\theta \end{pmatrix}$	A1
	Conclude, that by induction result is true for all positive integers	B1 cso (5)
(b)	When $n = 1$, $\mathrm{f}(n) = 7\times5-3 = 32$, which is divisible by 16, so result true for $n = 1$	B1
	Consider $\mathrm{f}(k+1) - \mathrm{f}(k) = (4k+7)5^{k+1} - (4k+3)5^k$	M1
	$= 5^k(20k+35-4k-3)$	M1
	$= 5^k(16k+32)$, which is divisible by 16	A1
	If f(k) is divisible by 16, then this implies f(k+1) is also divisible by 16 Thus by induction f(n) is divisible by 16 for all positive integers n.	B1 cso (5)
		[10]

Question Number	Scheme	Marks
Q7 (a)	If the lines meet, $-1+3\lambda=-4+3\mu$ and $2+4\lambda=2\mu$	M1
	Solve to give $\lambda=0(\mu=1)$	M1 A1
	Also $1-\lambda=\alpha$ and so $\alpha=1$.	B1 (4)
(b)	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -1 & 3 & 4\\ 0 & 3 & 2\end{vmatrix}=\begin{pmatrix}-6\\2\\-3\end{pmatrix}$	M1 A1
	$\boldsymbol{r}\bullet\begin{pmatrix}-6\\2\\-3\end{pmatrix}=\begin{pmatrix}-6\\2\\-3\end{pmatrix}\bullet(e.g.\begin{pmatrix}1\\-1\\2\end{pmatrix})=-14$	M1
	Hence $-6x+2y-3z+14=0$	A1 (4)
(c)	$\pm(\mathbf{a}_1-\mathbf{a}_2)=\pm(\mathbf{i}-3\mathbf{j}-2\mathbf{k})$	M1
	$\left\lvert\dfrac{(\mathbf{a}_1-\mathbf{a}_2)\bullet\mathbf{n}}{\lvert\mathbf{n}\rvert}\right\rvert=\left\lvert\dfrac{(\mathbf{i}-3\mathbf{j}-2\mathbf{k})\bullet(-6\mathbf{i}+2\mathbf{j}-3\mathbf{k})}{\lvert-6\mathbf{i}+2\mathbf{j}-3\mathbf{k}\rvert}\right\rvert=\left\lvert\dfrac{-6-6+6}{\sqrt{6^2+2^2+3^2}}\right\rvert$	M1
	Distance is $\dfrac{6}{7}$	A1 cso (3)
		[11]

Question Number	Scheme	Marks
Q8 (a)	$\sqrt{\{(x-3)^2+y^2\}}=2\sqrt{\{x^2+(y-4)^2\}}$ or $(x-3)^2+y^2=4\{x^2+(y-4)^2\}$	M1
	$3x^2+3y^2+6x-32y+55=0$	A1
	$(x+1)^2+(y-\frac{16}{3})^2=\frac{100}{9}$	M1
	Centre is (–1, 16/3) and radius is 10/3	A1,A1,A1 cso
		(6)
(b)	$w=\frac{12}{z}\to z=\frac{12}{w}$, and so $\left\lvert\frac{12}{w}-3\right\rvert=2\left\lvert\frac{12}{w}-4\mathrm{i}\right\rvert$ substituting for z	M1
	$\lvert 3w-12\rvert=2\lvert 4\mathrm{i}w-12\rvert$ multiplication by $\lvert w\rvert$ or equivalent	M1
	$\lvert w-4\rvert=\frac{8}{3}\lvert w+3\mathrm{i}\rvert$ obtains the locus of Q in the required form	M1, A2, 1, 0
	A2 if completely correct deduct 1 for each error on their a, k or b	
		(5)
		[11]

June 2009
6677 Mechanics M1
Mark Scheme

Question Number	Scheme	Marks
Q1		
	$45 = 2u + \frac{1}{2}a2^2 \quad \Rightarrow \quad 45 = 2u + 2a$	M1 A1
	$165 = 6u + \frac{1}{2}a6^2 \quad \Rightarrow \quad 165 = 6u + 18a$	M1 A1
	eliminating either u or a	M1
	$u = 20$ and $a = 2.5$	A1 A1 **[7]**
Q2 (a)	$\tan\theta = \frac{p}{2p} \Rightarrow \theta = 26.6°$	M1 A1 (2)
(b)	$\mathbf{R} = (\mathbf{i} - 3\mathbf{j}) + (p\mathbf{i} + 2p\mathbf{j}) = (1 + p)\mathbf{i} + (-3 + 2p)\mathbf{j}$	M1 A1
	$\mathbf{R}$ is parallel to $\mathbf{i} \Rightarrow (-3 + 2p) = 0$	DM1
	$\Rightarrow p = \frac{3}{2}$	A1 (4) **[6]**
Q3 (a)	For A: $-\dfrac{7mu}{2} = 2m(v_A - 2u)$	M1 A1
	$v_A = \dfrac{u}{4}$	A1 (3)
(b)	For B: $\dfrac{7mu}{2} = m(v_B - -3u)$	M1 A1
	$v_B = \dfrac{u}{2}$	A1 (3)
	OR CLM:	OR
	$4mu - 3mu = 2m\dfrac{u}{4} + mv_B$	M1 A1
	$v_B = \dfrac{u}{2}$	A1 (3)
		[6]

Question Number	Scheme	Marks
Q4	$0.5g\sin\theta - F = 0.5a$	M1 A1 A1
	$F = \frac{1}{3}R$ seen	B1
	$R = 0.5g\cos\theta$	M1 A1
	Use of $\sin\theta = \frac{4}{5}$ or $\cos\theta = \frac{3}{5}$ or decimal equiv or decimal angle e.g 53.1° or 53°	B1
	$a = \dfrac{3g}{5}$ or 5.88 m s^{-2} or 5.9 m s^{-2}	DM1 A1 **[9]**
Q5	$F = P\cos 50^{o}$	M1 A1
	$F = 0.2R$ seen or implied.	B1
	$P\sin 50^{o} + R = 15g$	M1 A1 A1
	Eliminating R; Solving for P; $P =$ 37 (2 SF)	DM1;D M1; A1 **[9]**
Q6 (a)	For whole system: $1200 - 400 - 200 = 1000a$	M1 A1
	$a = 0.6\text{ m s}^{-2}$	A1 (3)
(b)	For trailer: $T - 200 = 200 \text{ x } 0.6$	M1 A1 **ft**
	$T = 320$ N	A1
		OR:
	OR: For car: $1200 - 400 - T = 800 \text{ x } 0.6$	M1 A1 **ft**
	$T = 320$ N	A1 (3)
(c)	For trailer: $200 + 100 = 200f$ or $-200f$	M1 A1
	$f = 1.5\text{ m s}^{-2}$ (-1.5)	A1
	For car: $400 + F - 100 = 800f$ or $-800f$	M1 A2
	$F = 900$	A1 (7)
	(N.B. For both: $400 + 200 + F = 1000f$)	**[13]**

Question Number	Scheme	Marks
Q7 (a)	$M(Q),\quad 50g(1.4-x)+20g \times 0.7 = T_P \times 1.4$	M1 A1
	$T_P = 588 - 350x$ Printed answer	A1 (3)
(b)	$M(P),\quad 50gx+20g \times 0.7 = T_Q \times 1.4$ or $R(\uparrow),\ T_P + T_Q = 70g$	M1 A1
	$T_Q = 98 + 350x$	A1 (3)
(c)	Since $0 < x < 1.4$, $98 < T_P < 588$ and $98 < T_Q < 588$	M1 A1 A1 (3)
(d)	$98+350x = 3(588-350x)$	M1
	$x = 1.19$	DM1 A1 (3) **[12]**
Q8 (a)	$\lvert \mathbf{v} \rvert = \sqrt{1.2^2+(-0.9)^2} = 1.5 \text{ m s}^{-1}$	M1 A1 (2)
(b)	$(\mathbf{r}_H =)100\mathbf{j} + t(1.2\mathbf{i} - 0.9\mathbf{j})$ m	M1 A1 (2)
(c)	$(\mathbf{r}_K =)9\mathbf{i} + 46\mathbf{j} + t(0.75\mathbf{i} + 1.8\mathbf{j})$ m	M1 A1
	$\overrightarrow{HK} = \mathbf{r}_K - \mathbf{r}_H = (9-0.45t)\mathbf{i} + (2.7t-54)\mathbf{j}$ m **Printed Answer**	M1 A1 (4)
(d)	Meet when $\overrightarrow{HK} = \mathbf{0}$	
	$(9-0.45t) = 0$ and $(2.7t-54) = 0$	M1 A1
	$t = 20$ from both equations	A1
	$\mathbf{r}_K = \mathbf{r}_H = (24\mathbf{i} + 82\mathbf{j})$ m	DM1 A1 **cso**
		(5)
		[13]

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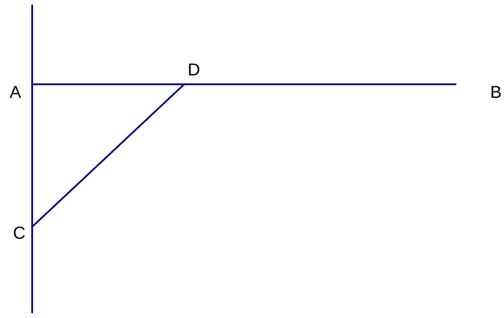
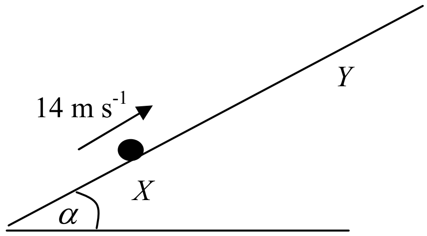
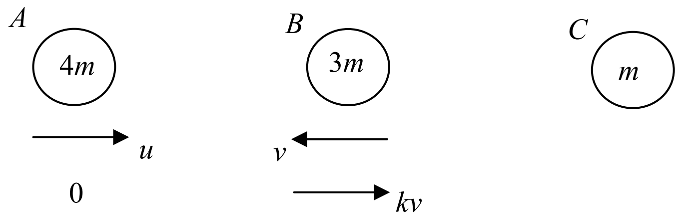
June 2009
6678 Mechanics M2
Mark Scheme

Question Number	Scheme	Marks
Q1	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	
	$5\mathbf{i} - 3\mathbf{j} = \frac{1}{4}\mathbf{v} - \frac{1}{4}(3\mathbf{i} + 7\mathbf{j})$	M1A1
	$\mathbf{v} = 23\mathbf{i} - 5\mathbf{j}$	A1
	$\lvert\mathbf{v}\rvert = \sqrt{23^2 + 5^2} = 23.5$	M1A1
		[5]
Q2 (a)	$\frac{dv}{dt} = 8 - 2t$	M1
	$8 - 2t = 0$	M1
	Max $v = 8 \times 4 - 4^2 = 16\,(\text{ms}^{-1})$	M1A1
		(4)
(b)	$\int 8t - t^2 dt = 4t^2 - \frac{1}{3}t^3 (+c)$	M1A1
	(t=0, displacement = 0 $\Rightarrow$ c=0)	
	$4T^2 - \frac{1}{3}T^3 = 0$	DM1
	$T^2(4 - \frac{T}{3}) = 0 \Rightarrow T = 0, 12$	DM1
	$T = 12$ (seconds)	A1
		(5) **[9]**
Q3 (a)	Constant v $\Rightarrow$ driving force = resistance	
	$\Rightarrow$ F=120 (N)	M1
	$\Rightarrow$ P=120 x 10 = 1200W	M1
		(2)
(b)	Resolving parallel to the slope, zero acceleration:	
	$\frac{P}{v} = 120 + 300g\sin\theta (= 330)$	M1A1A1
	$\Rightarrow \text{v} = \frac{1200}{330} = 3.6\,(\text{ms}^{-1})$	A1
		(4) **[6]**

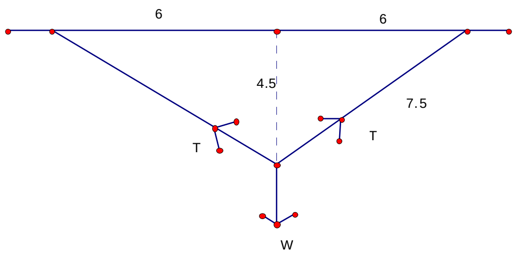
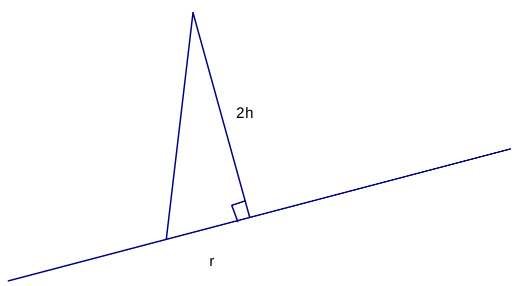
Question Number	Scheme	Marks
Q4 (a)	 Taking moments about A: $3g \times 0.75 = \frac{T}{\sqrt{2}} \times 0.5$	M1A1A1
	$T = 3\sqrt{2}g \times \frac{7.5}{5} = \frac{9\sqrt{2}g}{2} (= 62.4N)$	A1
		(4)
(b)	$\leftarrow \pm H = \frac{T}{\sqrt{2}} (= \frac{9g}{2} \approx 44.1N)$	B1
	$\uparrow \ \pm V + \frac{T}{\sqrt{2}} = 3g \ \ (\Rightarrow V = 3g - \frac{9g}{2} = \frac{-3g}{2} \approx -14.7\text{N})$	M1A1
	$\Rightarrow \ \lvert R \rvert = \sqrt{81+9} \times \frac{g}{2} \approx 46.5(N)$	M1A1
	at angle $\tan^{-1}\frac{1}{3}$ = 18.4° (0.322 radians) below the line of BA 161.6° (2.82 radians) below the line of AB (108.4° or 1.89 radians to upward vertical)	M1A1
		(7) **[11]**
Q5 (a)	Ratio of areas triangle:sign:rectangle = 1 : 5 : 6 (1800:9000:10800)	B1
	Centre of mass of the triangle is 20cm down from *AD* (seen or implied)	B1
	$\Rightarrow 6 \times 45 - 1 \times 20 = 5 \times \bar{y}$	M1A1
	$\bar{y} = 50cm$	A1
		(5)
(b)	Distance of centre of mass from *AB* is 60cm	B1
	Required angle is $\tan^{-1}\frac{60}{50}$ (their values)	M1A1ft
	$= 50.2^{\circ}$ (0.876 rads)	A1
		(4) **[9]**

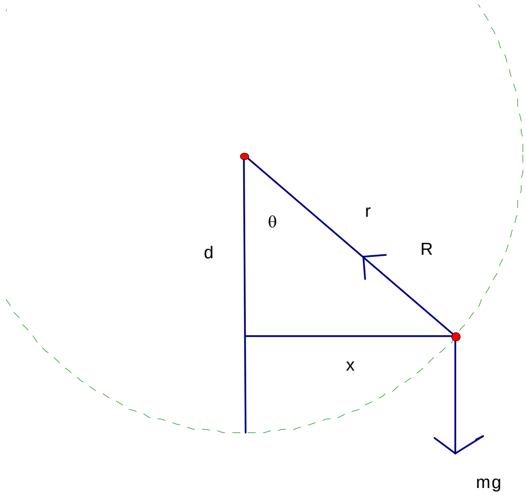
Question Number	Scheme	Marks
Q6 (a)	$\rightarrow \quad x = u\cos\alpha t = 10$	M1A1
	$\uparrow \quad y = u\sin\alpha t - \frac{1}{2}gt^2 = 2$	M1A1
	$\Rightarrow t = \frac{10}{u\cos\alpha}$	
	$2 = u\sin\alpha \times \frac{10}{u\cos\alpha} - \frac{g}{2} \times \frac{100}{u^2\cos^2\alpha}$	M1
	$= 10\tan\alpha - \frac{50g}{u^2\cos^2\alpha}$ (given answer)	A1
		(6)
(b)	$2 = 10 \times 1 - \frac{100g \times 2}{2u^2 \times 1}$	M1A1
	$u^2 = \frac{100g}{8}$, $u = \sqrt{\frac{100g}{8}} = 11.1$ (m s^{-1})	A1
	$\frac{1}{2}mu^2 = m \times 9.8 \times 2 + \frac{1}{2}mv^2$	M1A1
	$v = 9.1 ms^{-1}$	A1
		(6)
		[12]

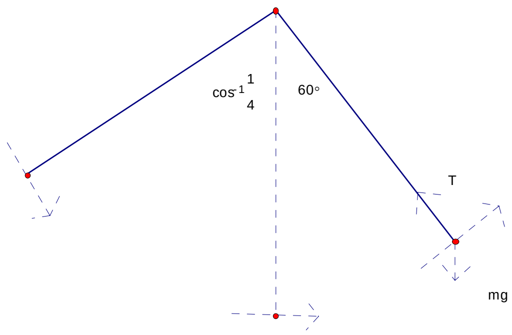
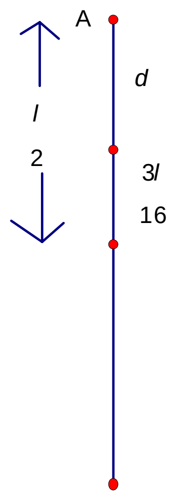
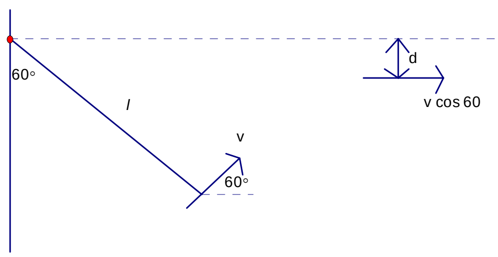
Question Number	Scheme	Marks
Q7 (a)		
	KE at $X = \frac{1}{2}mv^2 = \frac{1}{2}\times 2\times 14^2$	B1
	GPE at $Y =$ $mgd\sin\alpha \left(= 2\times g\times d\times\frac{7}{25}\right)$	B1 B1
	Normal reaction $R = mg\cos\alpha$	M1
	Friction = $\mu\times R = \frac{1}{8}\times 2g\times\frac{24}{25}$	M1A1
	Work Energy: $\frac{1}{2}mv^2 - mgd\sin\alpha = \mu\times R\times d$ or equivalent	
	$196 = \frac{14gd}{25} + \frac{6gd}{25} = \frac{20gd}{25}$	A1
	$d = 25$ m	(7)
(b)	Work Energy	
	First time at X: $\frac{1}{2}mv^2 = \frac{1}{2}m14^2$	
	Work done = $\mu\times R\times 2d = \frac{1}{8}\times 2g\times\frac{24}{25}\times 2d$	
	Return to X: $\frac{1}{2}mv^2 = \frac{1}{2}m14^2 - \frac{1}{8}\times 2g\times\frac{24}{25}\times 50$	M1A1
	$v = 8.9\ \text{ms}^{-1}$ (accept $8.85\ \text{ms}^{-1}$)	DM1A1
		(4)
	OR: Resolve parallel to XY to find the acceleration and use of $v^2 = u^2 + 2as$	
	$2a = 2g\sin\alpha - F_{max} = 2g\times\frac{7}{25} - \frac{6g}{25} = \frac{8g}{25}$	M1A1
	$v^2 = (0+)2\times a\times s = 8g$; $v = 8.9$ (accept $8.85\ \text{ms}^{-1}$)	DM1;A1
		[11]

Question Number	Scheme	Marks
Q8 (a)		
	Conservation of momentum: $4mu - 3mv = 3mkv$	M1A1
	Impact law: $kv = \frac{3}{4}(u + v)$	M1A1
	Eliminate k: $4mu - 3mv = 3m \times \frac{3}{4}(u + v)$	DM1
	$u = 3v$ (Answer given)	A1
		(6)
(b)	$kv = \frac{3}{4}(3v + v)$, $k = 3$	M1,A1
		(2)
(c)	Impact law: $(kv + 2v)e = v_C - v_B$ $(5ve = v_C - v_B)$	B1
	Conservation of momentum : $3 \times kv - 1 \times 2v = 3v_B + v_c$ $(7v = 3v_B + v_c)$	B1
	Eliminate v_C : $v_B = \frac{v}{4}(7 - 5e) > 0$ hence no further collision with A.	M1 A1
		(4)
		[12]

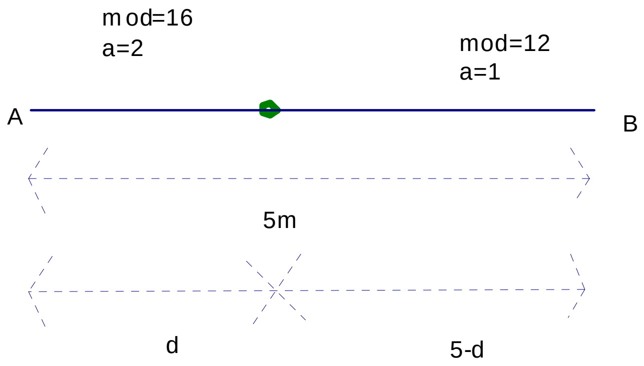
June 2009
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	 Resolving vertically: $2T\cos\theta = W$	M1A2,1,0
	Hooke's Law: $T = \dfrac{80\times 3.5}{4}$	M1A1
	$W = 84\text{N}$	A1
(b)	$\text{EPE} = 2\times\dfrac{80\times 3.5^2}{2\times 4}, = 245$ (or awrt 245)	M1A1ft,A1
	(alternative $\dfrac{80\times 7^2}{16} = 245$)	
		[9]
Q2 (a)	Object / Mass / c of m above base Cone / m / $2h+3h$ Base / $3m$ / h Marker / $4m$ / d	B1(ratio masses) B1(distances)
	m x $5h$ +$3m$ x h = $4m$ x d	M1A1ft
	$d = 2h$	A1
(b)	 $\dfrac{r}{d} = \dfrac{1}{12}$	M1A1ft
	$6r = h$	A1
		[8]

Question Number	Scheme	Marks
Q3 (a)	 $\leftrightarrow \quad R\sin\theta = mx\omega^2$	M1 A1
	$R\times\dfrac{x}{r} = mx\times\dfrac{3g}{2r}$	M1
	$R = \dfrac{3mg}{2}$	A1
(b)	$\updownarrow \quad R\cos\theta = mg$	M1 A1
	$\dfrac{3mg}{2}\times\dfrac{d}{r} = mg$	M1
	$d = \dfrac{2}{3}r$	A1
		[8]
Q4 (a)	Volume $= \int_{\frac{1}{4}}^{1} \pi y^2\,dx = \int_{\frac{1}{4}}^{1} \pi\dfrac{1}{x^4}dx$	M1A1
	$= \left[\pi\times\dfrac{-1}{3x^3}\right]_{\frac{1}{4}}^{1}$	A1ft
	$= \pi(\dfrac{-1}{3}+\dfrac{64}{3}) = 21\pi$ *	A1
(b)	$21\pi\rho\bar{x} = \rho\int \pi y^2 x\,dx = \rho\int \pi\dfrac{1}{x^4}x\,dx$	M1A1
	$21\pi\bar{x} = \pi\left[\dfrac{-1}{2x^2}\right]_{\frac{1}{4}}^{1}$	A1ft
	$\bar{x} = \dfrac{1}{21}(\dfrac{-1}{2}+\dfrac{16}{2}) = \dfrac{5}{14}$ or awrt 0.36	A1
	$\bar{y} = 0$ by symmetry	B1
		[9]

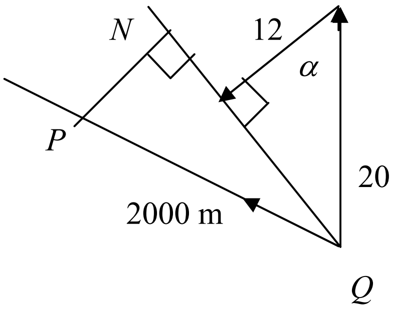
Question Number	Scheme	Marks
Q5 (a)		
	Energy:	
	$(\frac{1}{2}mu^2+)mgl(\cos\theta-\frac{1}{4})=\frac{1}{2}mv^2$	M1A1
	Resolving:	
	$T-mg\cos\theta=\frac{mv^2}{l}$	M1A1
	Eliminate v^2:	
	$T=mg\cos\theta+\frac{1}{l}(2mgl(\cos\theta-\frac{1}{4}))$	M1
	$T=3mg\cos\theta-\frac{mg}{2}$ *	A1
(b)		
	$\theta=60°\Rightarrow mv^2=2mgl(\frac{1}{2}-\frac{1}{4})$	M1
	$\Rightarrow v^2=\frac{gl}{2}$	
	vertical motion under gravity:	
	$\updownarrow\ 0=(v\cos30°)^2-2gs$	M1
	$0=\frac{gl}{2}\times\frac{3}{4}-2gs\Rightarrow s=\frac{3l}{16}$	A1
	Distance below A $=\frac{l}{2}-\frac{3l}{16}=\frac{5l}{16}$	M1A1
		[11]
Alternative for end of (b) using energy		
	$\frac{1}{2}mv^2-mgl\cos60=\frac{1}{2}m(v\cos60)^2-mgd$	M1A1
	$\frac{gl}{4}-\frac{gl}{2}=\frac{gl}{4}\times\frac{1}{4}-gd$	M1
	$d=\frac{1-4+8}{16}l=\frac{5l}{16}$	A1

Question Number	Scheme	Marks
Q6 (a)	At max v, driving force = resistance	
	Driving force $= \frac{80}{v}$	B1
	$\Rightarrow \frac{80}{20} = k \times 20^2 \Rightarrow k = \frac{1}{100}$	M1A1
	F = ma $\Rightarrow 100a = \frac{80}{v} - kv^2 \quad (= \frac{8000 - v^3}{100v})$	M1
	* $\Rightarrow v\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{8000 - v^3}{10000v}$ *	A1
(b)	$\int_4^8 \frac{10000v^2}{8000 - v^3}\mathrm{d}v = \int_0^D 1\mathrm{d}x$	M1A1
	$D = \left[-\frac{10000}{3}\ln\left\|8000 - v^3\right\|\right]_4^8$	A1
	$= (-\frac{10000}{3}\ln\frac{7488}{7936}) = 193.7..... \approx 194\,\text{m}$ (accept 190)	M1 A1
(c)	$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{8000 - v^3}{10000v} \Rightarrow \int_0^T 1\mathrm{d}t = \int_4^8 \frac{10000v}{8000 - v^3}\mathrm{d}v$	M1A1
	$\Rightarrow T \approx \frac{1}{2} \times 2 \times 10000 \times \left\{\frac{4}{7936} + \frac{2 \times 6}{7784} + \frac{8}{7488}\right\}$	M1
	$\Rightarrow T(= 31.1409....) \approx 31$	A1
		[14]

Question Number	Scheme	Marks
Q7 (a)		
	Hooke's law: Equilibrium $\Rightarrow \frac{16(d-2)}{2} = \frac{12(4-d)}{1}$	M1A1A1
	$\Rightarrow d = 3.2$	A1
	so extensions are 1.2m and 0.8m.	A1
(b)	If the particle is displaced distance x towards ***B*** then	
	$-m\ddot{x} = \frac{16(1.2+x)}{2} - \frac{12(0.8-x)}{1} (= 20x)$	M1A1ft A1ft
	$\Rightarrow \ddot{x} = -40x$ or $\ddot{x} = -\frac{20}{m} (\Rightarrow \text{SHM})$	A1
(c)	$T = \frac{2\pi}{\sqrt{40}}$	B1ft
	$a = \frac{\sqrt{10}}{\text{their } \omega}$	B1ft
	$x = a \sin \omega t$ their a, their ω	M1
	$\frac{1}{4} = \frac{1}{2} \sin \sqrt{40} t$	A1
	$\sqrt{40} t = \frac{\pi}{6} (\Rightarrow t = \frac{\pi}{6\sqrt{40}})$	M1
	Proportion $\frac{4t}{T} = \frac{4\pi}{6\sqrt{40}} \times \frac{\sqrt{40}}{2\pi} = \frac{1}{3}$	M1A1
		[16]

June 2009
6680 Mechanics M4
Mark Scheme

Question Number	Scheme	Marks
Q1	CLM along plane: $v\cos 30^o = u\cos 45^o$	M1 A1
	$v = u\sqrt{\frac{2}{3}}$	A1
	Fraction of KE Lost $= \dfrac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \dfrac{\frac{1}{2}mu^2 - \frac{1}{2}m\frac{2}{3}u^2}{\frac{1}{2}mu^2} = \dfrac{1}{3}$	M1 M1 A1
		[6]
Q2	$-mg - mkv^2 = ma$	M1 A1
	$-(g+kv^2) = v\dfrac{\mathrm{d}v}{\mathrm{d}x}$	M1
	$\pm\int_0^X \mathrm{d}x = \int_{\sqrt{\frac{g}{k}}}^{\frac{1}{2}\sqrt{\frac{g}{k}}} \dfrac{-v\mathrm{d}v}{(g+kv^2)}$	DM1 A1 (both previous)
	$X = \dfrac{1}{2k}\left[\ln(g+kv^2)\right]_{\frac{1}{2}\sqrt{\frac{g}{k}}}^{\sqrt{\frac{g}{k}}}$	M1 A1
	$= \dfrac{1}{2k}\left(\ln 2g - \ln\dfrac{5g}{4}\right)$	M1
	$= \dfrac{1}{2k}\ln\dfrac{8}{5}$	A1
		[9]

Question Number	Scheme	Marks
Q3 (a)		M1
	$\cos\alpha = \frac{12}{20}$	M1 A1
	Bearing is $180^\circ + \alpha = 233^o$ (nearest degree)	A1
		(4)
(b)	$PN = 2000\cos(135^\circ - \alpha) = 200\sqrt{2}$ m or decimal equivalent	M1A1ft A1
		(3)
(c)	$\sqrt{20^2 - 12^2}$	B1
	Time to closest approach $= \dfrac{QN}{\sqrt{20^2 - 12^2}}$	M1
	$= \dfrac{2000\sin(135^\circ - \alpha)}{16}$	A1
	Distance moved by Q = their t x 12	DM1
	$= 1050\sqrt{2}$ m or decimal equivalent	A1
		(5)
		[12]

Question Number	Scheme	Marks
Q4 (a)	$V = -mg2a\sin 2\theta - \frac{7}{20}mg(L - 4a\sin\theta)$	M1 B1 A1
	$= \frac{1}{5}mga(7\sin\theta - 10\sin 2\theta) - \frac{7}{20}mgL$	A1
		(4)
(b)	$\frac{dV}{d\theta} = \frac{1}{5}mga(7\cos\theta - 20\cos 2\theta)$	M1 A1
	$\frac{1}{5}mga(7\cos\theta - 20\cos 2\theta) = 0$	DM1
	$7\cos\theta - 20(2\cos^2\theta - 1) = 0$	DM1
	$40\cos^2\theta - 7\cos\theta - 20 = 0$	A1
	$(5\cos\theta - 4)(8\cos\theta + 5) = 0$	
	$\cos\theta = \frac{4}{5}$ or ($\cos\theta = -\frac{5}{8} \Rightarrow 2\theta > 180^\circ$)	DM1 A1 DM1
		(8)
(c)	$\frac{d^2V}{d\theta^2} = \frac{1}{5}mga(-7\sin\theta + 40\sin 2\theta)$	M1 A1
	$= \frac{1}{5}mga(-7\sin\theta + 80\sin\theta\cos\theta)$	
	When $\cos\theta = \frac{4}{5}$,	
	$\frac{d^2V}{d\theta^2} = \frac{1}{5}mga(\frac{-21}{5} + 80 \text{x} \frac{3}{5} \text{x} \frac{4}{5}) = \frac{171}{25}mga$	M1
	> 0 therefore stable	A1 cso
		(4)
		[16]

Question Number	Scheme	Marks
Q5 (a)	CLM: $2(\mathbf{i} + 2\mathbf{j}) + -2\mathbf{i} = 2\mathbf{j} + \mathbf{v}$	M1 A1
	$\mathbf{v} = 2\mathbf{j}$ m s^{-1}	A1
		(3)
(b)	$\mathbf{I} = 2(\mathbf{j} - (\mathbf{i} + 2\mathbf{j}))$	M1 A1
	$= (-2\mathbf{i} - 2\mathbf{j})$ Ns	A1
	Since **I** acts along l.o.c.c. , l.o.c.c is parallel to $\mathbf{i} + \mathbf{j}$	B1
		(4)
(c)	Before A: $(\mathbf{i} + 2\mathbf{j}).\frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{3}{\sqrt{2}}$	
	B: $-2\mathbf{j}.\frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{-2}{\sqrt{2}}$	
	After A: $\mathbf{j}.\frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{1}{\sqrt{2}}$	M1 A3
	B: $2\mathbf{j}.\frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{2}{\sqrt{2}}$	
	NIL:	
	$e = \dfrac{\frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{\frac{3}{\sqrt{2}} - \frac{-2}{\sqrt{2}}} = \frac{1}{5}$	DM1 A1
		(6)
		[13]

Question Number	Scheme	Marks
Q6 (a)	$(\rightarrow),\ T = m\ddot{y}$	M1
	Hooke's Law:	
	$T = \dfrac{2mn^2ax}{2a} = mn^2x$	B1
	$x + y = \frac{1}{2}ft^2$ $\dot{x} + \dot{y} = ft$ $\ddot{x} + \ddot{y} = f$	B2
	$so, \quad (\rightarrow),\ mn^2x = m\ddot{y} = m(f - \ddot{x})$	DM1
	$\ddot{x} + n^2x = f$ **	A1
		(6)
(b)	C.F. : $x = A\cos nt + B\sin nt$	B1
	P.I. : $x = \dfrac{f}{n^2}$	B1
	Gen solution: $x = A\cos nt + B\sin nt + \dfrac{f}{n^2}$	M1
	$\dot{x} = -An\sin nt + Bn\cos nt$ follow their PI	M1 A1ft
	$t = 0, x = 0 \Rightarrow A = -\dfrac{f}{n^2}$ $t = 0, \dot{x} = 0 \Rightarrow B = 0$	M1 A1
	$x = \dfrac{f}{n^2}(1 - \cos nt)$	A1
		(8)
(c)	$\dot{x} = 0 \Rightarrow \quad nt = \pi$	M1
	$x_{\max} = \dfrac{f}{n^2}(1 - -1) = \dfrac{2f}{n^2}$	M1 A1
		(3)
(d)	$\dot{y} = ft - \dot{x}$	M1
	$= f\dfrac{\pi}{n} - 0 = \dfrac{f\pi}{n}$	A1
		(2)
		[19]

June 2009
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
Q1	$\pm(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$	B1
	$((4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})+(8\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})).(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}) = \frac{1}{2}3v^2$	M1 A1 f.t.
	$12 = v$	A1
	$\mathbf{v} = \dfrac{12}{\sqrt{8^2 + (-4)^2 + 8^2}}(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$	M1
	$\mathbf{v} = (8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})\ \text{ms}^{-1}$	DM1 A1
		[7]
Q2	C.F. is $\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t$	B1
	P.I. is $\mathbf{r} = \mathbf{p}e^{2t}$	B1
	$\dot{\mathbf{r}} = 2\mathbf{p}e^{2t}$ $\ddot{\mathbf{r}} = 4\mathbf{p}e^{2t}$	B1 ft
	$4\mathbf{p}e^{2t} + 4\mathbf{p}e^{2t} = \mathbf{j}e^{2t}$	M1
	so, (PI is) $\mathbf{r} = \frac{1}{8}\mathbf{j}e^{2t}$	A1
	GS is $\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$	A1 ft
	$t = 0,\ \mathbf{r} = \mathbf{i} + \mathbf{j} \Rightarrow\ \mathbf{i} + \mathbf{j} = \mathbf{A} + \frac{1}{8}\mathbf{j} \Rightarrow\ \mathbf{i} + \frac{7}{8}\mathbf{j} = \mathbf{A}$	DM1 A1
	$\dot{\mathbf{r}} = -2\mathbf{A}\sin 2t + 2\mathbf{B}\cos 2t + \frac{1}{4}\mathbf{j}e^{2t}$	M1A1
	$t = 0,\ \dot{\mathbf{r}} = 2\mathbf{i} \Rightarrow\ 2\mathbf{i} = 2\mathbf{B} + \frac{1}{4}\mathbf{j} \Rightarrow\ \mathbf{i} - \frac{1}{8}\mathbf{j} = \mathbf{B}$ $\mathbf{r} = (\mathbf{i} + \frac{7}{8}\mathbf{j})\cos 2t + (\mathbf{i} - \frac{1}{8}\mathbf{j})\sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$	A1
		[11]

Question Number	Scheme	Marks
Q3 (a)	$mv = (m+\delta m)(v+\delta v) - (-\delta m)(c-v)$	M1 A2
	$mv = mv + m\delta v + v\delta m + c\delta m - v\delta m$	
	$-m\delta v = c\delta m$	
	$\frac{\mathrm{d}v}{\mathrm{d}m} = -\frac{c}{m}$ *	DM1 A1
		(5)
(b)		
	$\frac{\mathrm{d}m}{\mathrm{d}t} = -m_0 k$	B1
	$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}m} \text{ x } \frac{\mathrm{d}m}{\mathrm{d}t}$	M1
	$= -\frac{c}{m} \text{ x } -m_0 k$	
	$= \frac{cm_0 k}{m_0(1-kt)}$	DM1
	$= \frac{ck}{(1-kt)}$	A1
		(4)
		[9]

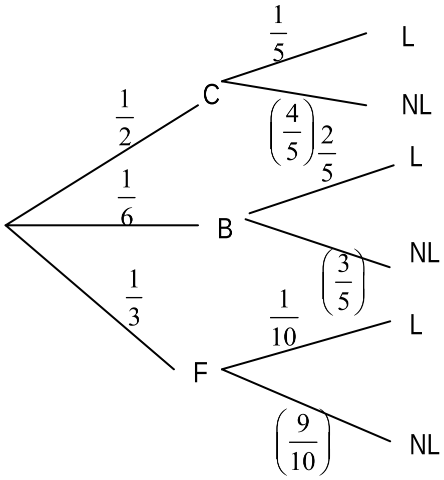
Question Number	Scheme	Marks
Q4 (a)		
	$\delta m = \dfrac{2Mx\delta x}{a^2}$	M1 A1
	$\delta I = \dfrac{1}{3}\dfrac{2Mx\delta x}{a^2}(2x)^2$	M1 A1
	$I = \displaystyle\int_0^a \dfrac{8Mx^3\mathrm{d}x}{3a^2}$	DM1
	$= \dfrac{8M}{3a^2}\left[\dfrac{x^4}{4}\right]_0^a$	
	$= \dfrac{2}{3}Ma^2$ *	A1
		(6)
(b)		
	$J.2a = \dfrac{2}{3}Ma^2\omega$	M1 A1
	$\dfrac{1}{2}\dfrac{2}{3}Ma^2\omega^2 = Mg\dfrac{2a}{3}(1+\cos 60^o)$	M1 A2
	solving for J	DM1
	$J = M\sqrt{\dfrac{ag}{3}}$	A1 (7)
		[13]

Question Number	Scheme	Marks
Q5 (a)	$(2\mathbf{i} + \mathbf{j}) + (-2\mathbf{j} - \mathbf{k}) + \mathbf{F}_3 = \mathbf{0}$	M1
	$\mathbf{F}_3 = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$	A1
	$\lvert\mathbf{F}_3\rvert = \sqrt{(-2)^2 + 1^2 + 1^2} = \sqrt{6}$ N	M1 A1 (4)
(b)	$(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \times (-2\mathbf{j} - \mathbf{k}) + (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \times (-2\mathbf{i} + \mathbf{j} + \mathbf{k})$	M1
	$(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + ((y - z)\mathbf{i} + (-2z - x)\mathbf{j} + (x + 2y)\mathbf{k})$	A3
	$y - z = -1$, $-x - 2z = -3$, $x + 2y = 1$	DM1
	$x = 1$, $y = 0$, $z = 1$ is a solution	DM1
	so, $\mathbf{r} = (\mathbf{i} + \mathbf{k}) + \lambda(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$ is a vector equn of line of action of $\mathbf{F}_3$	M1 A1 (8)
(c)	$(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \times (-2\mathbf{j} - \mathbf{k}) = \mathbf{G}$	M1
	$(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = (\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = \mathbf{G}$	A1
	$\lvert\mathbf{G}\rvert = \sqrt{1^2 + 3^2 + (-1)^2} = \sqrt{11}$ N m	M1 A1 (4)
		[16]

Question Number	Scheme	Marks
Q6 (a)		
	$\frac{1}{3}2m(4a)^2 + \frac{1}{12}4ma^2 + 4m(4a)^2$	B1 M1 A1
	$= \frac{32}{3}ma^2 + \frac{1}{3}ma^2 + 64ma^2$	
	$= 75ma^2$ *	A1
		(4)
(b)		
	$\frac{1}{2}75ma^2\omega^2 = 2mg2a(\cos\theta - \cos\alpha) + 4mg4a(\cos\theta - \cos\alpha)$	M1 A2
	$a\omega^2 = \frac{8}{15}g(\cos\theta - \frac{24}{25}) = \frac{8}{375}g(25\cos\theta - 24)$	A1
	$X - 6mg\cos\theta = 2m2a\omega^2 + 4m4a\omega^2 = 20ma\omega^2$	M1 A2
	$X = 6mg\cos\theta + 20m\frac{8}{375}g(25\cos\theta - 24)$	D M1
	$= \frac{50mg\cos\theta}{3} - \frac{256mg}{25}$	A1
		(9)
(c)		
	$-2mg2a\sin\theta - 4mg4a\sin\theta = 75ma^2\ddot{\theta}$	M1 A1
	$\ddot{\theta} = -\frac{4g}{15a}\sin\theta$	A1
	$\approx -\frac{4g}{15a}\theta$, SHM	M1
	$\text{Time} = \frac{1}{4}2\pi\sqrt{\frac{15a}{4g}}$	M1
	$= \frac{\pi}{4}\sqrt{\frac{15a}{g}}$	A1
		(6)
		[19]

June 2009
6683 Statistics S1
Mark Scheme

Question Number	Scheme		Marks
Q1 (a)	$(S_{pp}=)\ 38125 - \dfrac{445^2}{10}$		M1
	$=\ 18322.5$	awrt 18300	A1
	$(S_{pt}=)\ 26830 - \dfrac{445 \times 240}{10}$		
	$=\ 16150$	awrt 16200	A1
			(3)
(b)	$r = \dfrac{"16150"}{\sqrt{"18322.5" \times 21760}}$	Using their values for method	M1
	$= 0.8088...$	awrt 0.809	A1
			(2)
(c)	As the temperature increases the pressure increases.		B1
			(1)
			[6]
Notes	1(a) M1 for seeing a correct expression $38125 - \dfrac{445^2}{10}$ or $26830 - \dfrac{445 \times 240}{10}$ If no working seen, at least one answer must be exact to score M1 by implication. 1(b) Square root and their values with 21760 all in the right places required for method. Anything which rounds to (awrt) 0.809 for A1. 1(c) Require a correct statement in **context** using <u>temperature</u>/<u>heat</u> and <u>pressure</u> for B1. Don't allow " as t increases p increases". Don't allow proportionality. Positive correlation only is B0 since there is no interpretation.		

Question Number	Scheme		Marks
Q2 (a)		Correct tree All labels Probabilities on correct branches	B1 B1 B1 (3)
(b)(i)	$\frac{1}{3} \times \frac{1}{10} = \frac{1}{30}$ or equivalent		M1 A1 (2)
(ii)	$\text{CNL} + \text{BNL} + \text{FNL} = \frac{1}{2}\times\frac{4}{5}+\frac{1}{6}\times\frac{3}{5}+\frac{1}{3}\times\frac{9}{10}$		M1
	$= \frac{4}{5}$ or equivalent		A1 (2)
(c)	$P(F'/L) = \frac{P(F' \cap L)}{P(L)}$	Attempt correct conditional probability **but see notes**	M1
	$= \dfrac{\frac{1}{6}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{5}}{1-(ii)}$	$\dfrac{\text{numerator}}{\text{denominator}}$	$\dfrac{\text{A1}}{\text{A1ft}}$
	$= \dfrac{\frac{5}{30}}{\frac{1}{5}} = \frac{5}{6}$ or equivalent	cao	A1 (4) **[11]**
Notes	Exact decimal equivalents required throughout if fractions not used e.g. 2(b)(i) $0.0\dot{3}$ Correct path through their tree given in their probabilities award Ms 2(a) All branches required for first B1. Labels can be words rather than symbols for second B1. Probabilities from question enough for third B1 i.e. bracketed probabilities not required. Probabilities and labels swapped i.e. labels on branches and probabilities at end can be awarded the marks if correct. 2(b)(i) Correct answer only award both marks. 2(b)(ii) At least one correct path identified and attempt at adding all three multiplied pairs award M1 2(c) Require probability on numerator and division by probability for M1.Require numerator correct for their tree for M1. Correct formula seen and used, accept denominator as attempt and award M1 No formula, denominator must be correct for their tree or 1-(ii) for M1 1/30 on numerator only is M0, P(L/F') is M0.		

Question Number	Scheme	Marks
Q3 (a)	1(cm) cao	B1
(b)	10 cm^2 represents 15 10/15 cm^2 represents 1 or 1cm² represents 1.5 Therefore frequency of 9 is $\frac{10}{15}\times 9$ or $\frac{9}{1.5}$ Require x $\frac{2}{3}$ or $\div 1.5$ height = 6(cm)	 M1 A1 **[3]**
Notes	If 3(a) and 3(b) incorrect, but their (a) x their (b)=6 then award B0M1A0 3(b) Alternative method: f/cw=15/6=2.5 represented by 5 so factor x2 award M1 So f/cw=9/3=3 represented by 3x2=6. Award A1.	

Question Number	Scheme	Marks
Q4 (a)	$Q_2 = 17 + \left(\frac{60-58}{29}\right) \times 2$	M1
	$= 17.1$ (17.2 if use 60.5) awrt 17.1 (or17.2)	A1
		(2)
(b)	$\sum fx = 2055.5$ $\sum fx^2 = 36500.25$ Exact answers can be seen below or implied by correct answers.	B1 B1
	Evidence of attempt to use midpoints with at least one correct	M1
	Mean = 17.129… awrt 17.1	B1
	$\sigma = \sqrt{\frac{36500.25}{120} - \left(\frac{2055.5}{120}\right)^2}$	M1
	$= 3.28$ ($s = 3.294$) awrt 3.3	A1
		(6)
(c)	$\frac{3(17.129 - 17.1379...)}{3.28} = -0.00802$ Accept 0 or awrt 0.0	M1 A1
	No skew/ slight skew	B1
		(3)
(d)	The skewness is very small. Possible.	B1 B1dep
		(2)
		[13]
Notes	4(a) Statement of $17 + \frac{\text{freq into class}}{\text{class freq}} \times \text{cw}$ and attempt to sub or $\frac{m-17}{19-17} = \frac{60(.5)-58}{87-58}$ or equivalent award M1 cw=2 or 3 required for M1. 17.2 from cw=3 award A0. 4(b) Correct $\sum fx$ and $\sum fx^2$ can be seen in working for both B1s Midpoints seen in table and used in calculation award M1 Require complete correct formula including use of square root and attempt to sub for M1. No formula stated then numbers as above or follow from (b) for M1 $(\sum fx)^2, \sum (fx)^2 or \sum f^2x$ used instead of $\sum fx^2$ in sd award M0 Correct answers only with no working award 2/2 and 6/6 4(c) Sub in their values into given formula for M1 4(d) No skew / slight skew / 'Distribution is almost symmetrical' / 'Mean approximately equal to median' or equivalent award first B1. Don't award second B1 if this is not the case. Second statement should imply 'Greg's suggestion that a normal distribution is suitable is possible' for second B1 dep. If B0 awarded for comment in (c).and (d) incorrect, allow follow through from the **comment** in (c).	

Question Number	Scheme	Marks
Q5 (a)	$b = \dfrac{59.99}{33.381}$	M1
	$= 1.79713.....$ 1.8 or awrt 1.80	A1
	$a = 32.7 - 1.79713... \times 51.83$	M1
	$= -60.44525...$ awrt -60	A1
	$w = -60.445251... + 1.79713...l$ l and w required and awrt 2sf	A1ft (5)
(b)	$w = -60.445251... + 1.79713... \times 60$	M1
	$= 47.3825...$ In range 47.3 – 47.6 inclusive	A1 (2)
(c)	It is extrapolating so (may be) unreliable.	B1, B1dep (2) **[9]**
Notes	5(a) Special case $b = \dfrac{59.99}{120.1} = 0.4995$ M0A0 $a = 32.7 - 0.4995 \times 51.83$ M1A1 $w = 6.8 + 0.50l$ at least 2 sf required for A1 5(b) Substitute into their answer for (a) for M1 5(c) 'Outside the range on the table' or equivalent award first B1	

Question Number	Scheme		Marks
Q6 (a)	$\begin{array}{\|c\|c\|c\|c\|}\hline 0 & 1 & 2 & 3 \\ \hline 3a & 2a & a & b \\ \hline\end{array}$		B1 (1)
(b)	$3a + 2a + a + b = 1$	or equivalent, using Sum of probabilities =1	M1
	$2a + 2a + 3b = 1.6$	or equivalent, using E(X)=1.6	M1
	$14a = 1.4$	Attempt to solve	M1dep
	$a = 0.1$	cao	B1
	$b = 0.4$	cao	B1 (5)
(c)	$\mathrm{P}(0.5 < x < 3) = \mathrm{P}(1) + \mathrm{P}(2)$	3a or their $2a$+their a	M1
	$= 0.2 + 0.1$		
	$= 0.3$	Require $0<3a<1$ to award follow through	A1 ft (2)
(d)	$\mathrm{E}(3X - 2) = 3\mathrm{E}(X) - 2$		M1
	$= 3 \times 1.6 - 2$		
	$= 2.8$	cao	A1 (2)
(e)	$\mathrm{E}(X^2) = 1\times0.2 + 4\times0.1 + 9\times0.4\ (= 4.2)$		M1
	$\mathrm{Var}(X) = \text{“}4.2\text{”} - 1.6^2$		M1
	$= 1.64$ **given answer**	cso	A1 (3)
(f)	$\mathrm{Var}(3X - 2) = 9\,\mathrm{Var}(X)$		M1
	$= 14.76$	awrt 14.8	A1 (2) **[15]**

Notes

6(a) Condone a clearly stated in text but not put in table.
6(b) Must be attempting to solve 2 different equations so third M dependent upon first two Ms being awarded.
Correct answers seen with no working B1B1 only, 2/5
Correctly verified values can be awarded M1 for correctly verifying sum of probabilities =1, M1 for using E(X)=1.6 M0 as no attempt to solve and B1B1 if answers correct.
6(d) 2.8 only award M1A1
6(e) Award first M for at least two non-zero terms correct. Allow first M for correct expression with a and b e.g. E(X^2) = $6a+9b$
Given answer so award final A1 for correct solution.
6(f) 14.76 only award M1A1

Question Number	Scheme		Marks
Q7(a) (i)	$P(A \cup B) = a + b$	cao	B1
(ii)	$P(A \cup B) = a + b - ab$	or equivalent	B1 (2)
(b)	$P(R \cup Q) = 0.15 + 0.35$ $= 0.5$	0.5	B1 (1)
(c)	$P(R \cap Q) = P(R\|Q) \times P(Q)$ $= 0.1 \times 0.35$		M1
	$= 0.035$	**0.035**	A1 (2)
(d)	$P(R \cup Q) = P(R) + P(Q) - P(R \cap Q)$ OR $P(R) = P(R \cap Q') + P(R \cap Q)$ $= 0.15 +$ their (c)		M1
	$0.5 = P(R) + 0.35 - 0.035$ $\quad = 0.15 + 0.035$ $P(R) = 0.185$ $\quad = 0.185$	0.185	A1 (2) **[7]**
Notes	7(a) (i) Accept $a + b - 0$ for B1 **Special Case** If answers to (i) and (ii) are (i) $P(A)+P(B)$ and (ii) $P(A)+P(B)-P(A)P(B)$ award B0B1 7(a)(i) and (ii) answers must be clearly labelled or in correct order for marks to be awarded.		

Question Number	Scheme	Marks
Q8 (a)	Let the random variable X be the lifetime in hours of bulb	
	$P(X < 830) = P(Z < \frac{\pm(830-850)}{50})$ Standardising with 850 and 50	M1
	$= P(Z < -0.4)$	
	$= 1 - P(Z < 0.4)$ Using 1-(probability>0.5)	M1
	$= 1 - 0.6554$	
	$= 0.3446$ or 0.344578 by calculator awrt 0.345	A1
		(3)
(b)	0.3446×500 Their (a) x 500	M1
	$= 172.3$ Accept 172.3 or 172 or 173	A1
		(2)
(c)	Standardise with 860 and σ and equate to z value $\frac{\pm(818-860)}{\sigma} = z$ value	M1
	$\frac{818-860}{\sigma} = -0.84(16)$ or $\frac{860-818}{\sigma} = 0.84(16)$ or $\frac{902-860}{\sigma} = 0.84(16)$ or equiv.	A1
	$\pm 0.8416(2)$	B1
	$\sigma = 49.9$ 50 or awrt 49.9	A1
		(4)
(d)	Company Y as the ***mean*** is greater for Y. both	B1
	They have (approximately) the same ***standard deviation*** or ***sd***	B1
		(2)
		[11]
Notes	8(a) If 1-z used e.g. 1-0.4=0.6 then award second M0 8(c) M1 can be implied by correct line 2 A1 for completely correct statement or equivalent. Award B1 if 0.8416(2) seen Do not award final A1 if any errors in solution e.g. negative sign lost. 8(d) Must use statistical terms as underlined.	

June 2009
6684 Statistics S2
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	$[X \sim \mathrm{B}(30, 0.15)]$	
	$\mathrm{P}(X \le 6)$, = 0.8474 awrt 0.847	M1, A1 (2)
(b)	$Y \sim \mathrm{B}(60, 0.15) \approx \mathrm{Po}(9)$ for using Po(9)	B1
	$\mathrm{P}(Y \le 12)$, = 0.8758 awrt 0.876	M1, A1 (3)
		[5]
	[N.B. normal approximation gives 0.897, exact binomial gives 0.894]	
(a)	M1 for a correct probability statement $\mathrm{P}(X \le 6)$ or $\mathrm{P}(X < 7)$ or $\mathrm{P}(X=0) + \mathrm{P}(X=1) + \mathrm{P}(X=2) + \mathrm{P}(X=4) + \mathrm{P}(X=5) + \mathrm{P}(X=6)$. (may be implied by long calculation) Correct answer gets M1 A1. allow 84.74%	
(b)	B1 may be implied by using Po(9). Common incorrect answer which implies this is 0.9261 M1 for a correct probability statement $\mathrm{P}(X \le 12)$ or $\mathrm{P}(X < 13)$ or $\mathrm{P}(X=0) + \mathrm{P}(X=1) + \ldots + \mathrm{P}(X=12)$ (may be implied by long calculation) and attempt to evaluate this probability using their Poisson distribution. Condone $\mathrm{P}(X \le 13) = 0.8758$ for B1 M1 A1 Correct answer gets B1 M1 A1 Use of normal or exact binomial get B0 M0 A0	

Question Number	Scheme	Marks
Q2	H_0: $\lambda = 2.5$ (or $\lambda = 5$) H1: $\lambda < 2.5$ (or $\lambda < 5$) λ or μ	B1B1
	$X \sim \text{Po}(5)$	M1
	$P(X \leq 1) = 0.0404$ or CR $X \leq 1$	A1
	[0.0404<0.05] this is significant or reject H_0 or it is in the critical region	M1
	There is evidence of a <u>decrease</u> in the (mean) <u>number/rate</u> of <u>deformed blood cells</u>	A1 (6) **[6]**
	1st B1 for H_0 must use lambda or mu; 5 or 2.5. 2nd B1 for H_1 must use lambda or mu; 5 or 2.5 1st M1 for use of Po(5) may be implied by probability(must be used not just seen) eg. P ($X = 1$) = 0.0404 - … would score M1 A0 1st A1 for 0.0404 seen or correct CR 2nd M1 for a correct statement (this may be contextual) comparing their probability and 0.05 (or comparing 1 with their critical region). Do not allow conflicting statements. 2nd A1 is not a follow through. Need the word decrease, number or rate and deformed blood cells for contextual mark. If they have used $\neq$ in H_1 they could get B1 B0 M1 A1 M1A0 mark as above except they gain the 1st A1 for $P(X \leq 1) = 0.0404$ or CR $X \leq 0$ 2nd M1 for a correct statement (this may be contextual) comparing their probability and 0.025 (or comparing 1 with their critical region) They may compare with 0.95 (one tail method) or 0.975 (one tail method) Probability is 0.9596.	

Question Number	Scheme	Marks
Q3 (a)	*A statistic* is a function of $X_1, X_2, \ldots X_n$ that does not contain any unknown parameters	B1 B1 (2)
(b)	The probability distribution of Y or the distribution of all possible values of Y (o.e.)	B1 (1)
(c)	Identify (ii) as not a statistic Since it contains unknown parameters μ and σ.	B1 dB1 (2)
		[5]
(a)	Examples of other acceptable wording: B1 e.g. is a function of the sample or the data / is a quantity calculated from the sample or the data / is a random variable calculated from the sample or the data B1 e.g. does not contain any unknown parameters/quantities contains only known parameters/quantities only contains values of the sample Y is a function of $X_1, X_2, \ldots X_n$ that does not contain any unknown parameters B1B1 is a function of the values of a sample with no unknowns B1B1 is a function of the sample values B1B0 is a function of all the data values B1B0 A random variable calculated from the sample B1B0 A random variable consisting of any function B0B0 A function of a value of the sample B1B0 A function of the sample which contains no other values/ parameters B1B0	
(b)	Examples of other acceptable wording All possible values of the statistic together with their associated probabilities	
(c)	1st B1 for selecting only (ii) 2nd B1 for a reason. This is dependent upon the first B1. Need to mention at least one of mu (mean) or sigma (standard deviation or variance) or unknown parameters. Examples since it contains mu B1 since it contains sigma B1 since it contains unknown parameters/quantities B1 since it contains unknowns B0	

Question Number	Scheme	Marks
Q4 (a)	$X \sim B(20, 0.3)$ $P(X \le 2) = 0.0355$ $P(X \le 9) = 0.9520$ so $P(X \ge 10) = 0.0480$ Therefore the critical region is $\{X \le 2\} \cup \{X \ge 10\}$	M1 A1 A1 A1A1 (5)
(b)	$0.0355 + 0.0480 = 0.0835$ awrt (0.083 or 0.084)	B1 (1)
(c)	11 is in the critical region there is evidence of a <u>change/ increase</u> in the <u>proportion/number</u> of <u>customers buying single tins</u>	B1ft B1ft (2)
		[8]
(a)	M1 for B(20,0.3) seen or used 1st A1 for 0.0355 2nd A1 for 0.048 3rd A1 for $(X) \le 2$ or $(X) < 3$ or [0,2] They get **A0** if they write $P(X \le 2/ X < 3)$ 4th A1 $(X) \ge 10$ or $(X) > 9$ or [10,20] They get **A0** if they write $P(X \ge 10/ X > 9)$ **10** $\le X \le 2$ etc is accepted To describe the critical regions they can use any letter or no letter at all. It does not have to be X.	
(b)	B1 correct answer only	
(c)	1st B1 for a correct statement about 11 and their critical region. 2nd B1 for a correct comment in context consistent with their CR and the value 11 Alternative solution 1st B0 $P(X \ge 11) = 1 - 0.9829 = 0.0171$ since no comment about the critical region 2nd B1 a correct contextual statement.	

Question Number	Scheme	Marks
Q5 (a)	X = the number of errors in 2000 words so $X \sim \text{Po}(6)$	B1
	$\text{P}(X \geq 4) = 1 - \text{P}(X \leq 3)$	M1
	$= 1 - 0.1512 \quad = 0.8488$ awrt 0.849	A1 (3)
(b)	Y = the number of errors in 8000 words. $Y \sim \text{Po}(24)$ so use a <u>Normal</u> approx	M1
	$Y \approx\sim \text{N}(24, \sqrt{24}^2)$	A1
	Require $\text{P}(Y \leq 20) = \text{P}\left(Z < \dfrac{20.5 - 24}{\sqrt{24}}\right)$	M1 M1
	$= \text{P}(Z < -0.714\ldots)$	A1
	$= 1 - 0.7611$	M1
	$= 0.2389$ awrt (0.237~0.239)	A1 (7)
	[N.B. Exact Po gives 0.242 and no $\pm$ 0.5 gives 0.207]	**[10]**

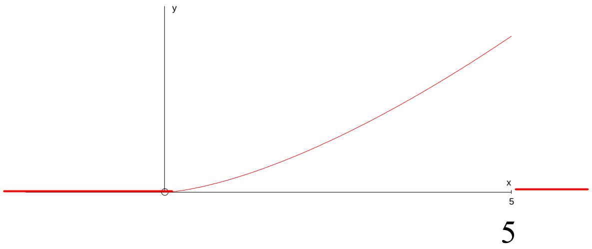
(a) B1 for seeing or using Po(6)
M1 for $1 - \text{P}(X \leq 3)$ or $1 - [\text{P}(X = 0) + \text{P}(X = 1) + \text{P}(X = 2) + \text{P}(X = 3)]$
A1 awrt 0.849

SC If B(2000, 0.003) is used and leads to awrt 0.849 allow B0 M1 A1
If no distribution indicated awrt 0.8488 scores B1M1A1 but any other awrt 0.849 scores B0M1A1

(b) 1st M1 for identifying the normal approximation
1st A1 for [mean = 24] **and** [sd = $\sqrt{24}$ or var = 24]

These first two marks may be given if the following are seen in the standardisation formula : 24
$\sqrt{24}$ or awrt 4.90

2nd M1 for attempting a continuity correction (20/ 28 $\pm$ 0.5 is acceptable)
3rd M1 for standardising using their mean and their standard deviation.
2nd A1 correct z value awrt $\pm$0.71 **or** this may be awarded if see $\dfrac{20.5 - 24}{\sqrt{24}}$ or $\dfrac{27.5 - 24}{\sqrt{24}}$
4th M1 for 1 - a probability from tables (must have an answer of < 0.5)
3rd A1 answer awrt 3 sig fig in range 0.237 – 0.239

Question Number	Scheme	Marks
Q6 (a)	$P(A>3) = \frac{2}{5} = 0.4$	B1 (1)
(b)	$(0.4)^3$,$= 0.064$ or $\dfrac{8}{125}$	M1, A1 (2)
(c)	$f(y) = \dfrac{d}{dy}(F(y)) = \begin{cases} \dfrac{3y^2}{125} & 0 \le y \le 5 \\ 0 & \textit{otherwise} \end{cases}$	M1A1 (2)
(d)	 Shape of curve and start at (0,0) Point (5, 0) labelled and curve between 0 and 5 and pdf ≥ 0	B1 B1 (2)
(e)	Mode = 5	B1 (1)
(f)	$E(Y) = \int_0^5 \left(\dfrac{3y^3}{125}\right) dy = \left[\dfrac{3y^4}{500}\right]_0^5 = \dfrac{15}{4}$ or 3.75	M1M1A1 (3)
(g)	$P(Y>3) = \begin{cases} \int_3^5 \frac{3y^2}{125}\, dy \\ \text{or } 1 - F(3) \end{cases} = 1 - \dfrac{27}{125} = \dfrac{98}{125} = 0.784$	M1A1 (2) **[13]**
(a) (b) (c) (d) (e) (f) (g)	B1 correct answer only(cao). Do not ignore subsequent working M1 for cubing their answer to part (a) A1 cao M1 for attempt to differentiate the cdf. They must decrease the power by 1 A1 fully correct answer including 0 otherwise. Condone < signs B1 for shape. Must curve the correct way and start at (0,0). No need for y = 0 (patios) lines B1 for point (5,0) labelled and pdf only existing between 0 and 5, may have y=0 (patios) for other values B1 cao 1st M1 for attempt to integrate their $yf(y)$ $y^n \to y^{n+1}$. 2nd M1 for attempt to use correct limits A1 cao M1 for attempt to find $P(Y > 3)$. e.g. writing $\int_3^5 \textit{their } f(y)$ must have correct limits or writing 1 – F(3)	

Question Number	Scheme	Marks
Q7 (a)	$\text{E}(X) = 2$ (by symmetry)	B1 (1)
(b)	$0 \le x < 2$, gradient $= \frac{\frac{1}{2}}{2} = \frac{1}{4}$ and equation is $y = \frac{1}{4}x$ so $a = \frac{1}{4}$	B1
	$b - \frac{1}{4}x$ passes through (4, 0) so $b = 1$	B1 (2)
(c)	$\text{E}(X^2) = \int_0^2 \left(\frac{1}{4}x^3\right) \text{d}x + \int_2^4 \left(x^2 - \frac{1}{4}x^3\right) \text{d}x$	M1M1
	$= \left[\frac{x^4}{16}\right]_0^2 + \left[\frac{x^3}{3} - \frac{x^4}{16}\right]_2^4$	A1
	$= 1 + \frac{64-8}{3} - \frac{256-16}{16} \quad = \quad 4\frac{2}{3}$ or $\frac{14}{3}$	M1A1
	$\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2 = \frac{14}{3} - 2^2$, $= \frac{2}{3}$ (so $\sigma = \sqrt{\frac{2}{3}} = 0.816$) **(*)**	M1 A1cso (7)
(d)	$\text{P}(X \le q) = \int_0^q \frac{1}{4}x \text{ d}x = \frac{1}{4}$, $\quad \frac{q^2}{2} = 1$ so $q = \sqrt{2} = 1.414$ awrt 1.41	M1A1,A1 (3)
(e)	2- σ= 1.184 so 2 - σ, 2 + σ is wider than IQR, therefore greater than 0.5	M1,A1 (2) **[15]**

(a) B1 cao

(b) B1 for value of *a*. B1 for value of *b*

(c) 1st M1 for attempt at $\int ax^3$ using their *a*. For attempt they need x^4. Ignore limits.

2nd M1 for attempt at $\int bx^2 - ax^3$ use their *a* and *b*. For attempt need to have either x^3 or x^4. Ignore limits

1st A1 correct integration for both parts

3rd M1 for use of the correct limits on each part

2nd A1 for either getting 1 and $3\frac{2}{3}$ or awrt 3.67 somewhere or $4\frac{2}{3}$ or awrt 4.67

4th M1 for use of $\text{E}(X^2) - [\text{E}(X)]^2$ must add both parts for E (X^2) and only have subtracted the mean^2 once. You must see this working

3rd A1 $\sigma = \sqrt{\frac{2}{3}}$ or $\sqrt{0.66667}$ or better with no incorrect working seen.

(d) M1 for attempting to find LQ, integral of either part of f(x) with their 'a' and 'b' = 0.25

Or their F(x) = 0.25 i.e. $\frac{ax^2}{2} = 0.25$ or $bx - \frac{ax^2}{2} + 4a - 2b = 0.25$ with their *a* and *b*

If they add both parts of their F(x), then they will get M0.

1st A1 for a correct equation/expression using their 'a'

2nd A1 for $\sqrt{2}$ or awrt 1.41

(e) M1 for a reason based on their quartiles

- Possible reasons are P(2 - σ<X< 2 + σ)= 0.6498 allow awrt 0.65
- 1.184 < LQ(1.414)

A1 for correct answer > 0.5

NB you must check the reason and award the method mark. A correct answer without a correct reason gets M0 A0

Question Number	Scheme	Marks
Q8 (a)	$X \sim \text{Po}(2)$ $\quad$ $P(X=4) = \dfrac{e^{-2} \times 2^4}{4!} = 0.0902$ $\quad$ awrt 0.09	M1 A1 (2)
(b)	$Y \sim \text{Po}(8)$ $P(Y > 10) = 1 - P(Y \le 10) = 1 - 0.8159 = 0.18411\ldots$ $\quad$ awrt 0.184	B1 M1A1 (3)
(c)	F = no. of faults in a piece of cloth of length x $\quad$ $F \sim \text{Po}(x \times \frac{2}{15})$ $e^{-\frac{2x}{15}} = 0.80$ $e^{-\frac{2}{15} \times 1.65} = 0.8025\ldots, \quad e^{-\frac{2}{15} \times 1.75} = 0.791\ldots$ These values are either side of 0.80 therefore $x = 1.7$ to 2 sf	M1A1 M1 A1 (4)
(d)	Expected number with no faults $= 1200 \times 0.8 = 960$ Expected number with some faults $= 1200 \times 0.2 = 240$ So expected profit $= 960 \times 0.60 - 240 \times 1.50, \quad = £216$	M1 A1 M1, A1 (4) **[13]**
(a)	M1 for use of Po(2) may be implied A1 awrt 0.09	
(b)	B1 for Po(8) seen or used M1 for $1 - P(Y \le 10)$ oe A1 awrt 0.184	
(c)	1st M1 for forming a suitable Poisson distribution of the form $e^{-\lambda} = 0.8$ 1st A1 for use of lambda as $\dfrac{2x}{15}$ (this may appear after taking logs) 2nd M1 for attempt to consider a range of values that will prove 1.7 is correct **OR** for use of logs to show lambda = … 2nd A1 correct solution only. Either get 1.7 from using logs or stating values either side	
S.C	for $e^{-\frac{2}{15} \times 1.7} = 0.797\ldots \approx 0.80 \quad \therefore x = 1.7$ to 2 sf allow 2nd M1A0	
(d)	1st M1 for one of the following 1200 p or 1200 (1 – p) where p = 0.8 or 2/15. 1st A1 for both expected values being correct or two correct expressions. 2nd M1 for an attempt to find expected profit, must consider with and without faults 2nd A1 correct answer only.	

June 2009
6691 Statistics S3
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	Randomly select a number between 00 and 499 (001 and 500) select every 500th person	B1 B1 (2)
(bi)	<u>Quota</u> **Advantage:** <u>Representative</u> sample can be achieved (with small sample size) <u>Cheap</u> (costs kept to a minimum) **<u>not</u>** "quick" Administration relatively <u>easy</u> **Disadvantage** Not possible to estimate sampling errors (due to lack of randomness) Not a random process Judgment of interviewer can affect choice of sample – <u>bias</u> Non-response not recorded Difficulties of defining controls e.g. social class	B1 B1 (2)
(bii)	<u>Systematic</u> **Advantage:** <u>Simple</u> or <u>easy</u> to use **<u>not</u>** "quick" or "cheap" or "efficient" It is suitable for large <u>samples</u> (not populations) **Disadvantage** Only random if the ordered list is (truly) random Requires a list of the population <u>or</u> must assign a number to each member of the pop.	B1 B1 (2) **[6]**
(a)	1st B1 for idea of using random numbers to select the first from1 - 500 (o.e.) 2nd B1 for selecting every 500th (name on the list) If they are clearly trying to carry out <u>stratified</u> sample then score B0B0	
(b)	Score B1 for any one line	
(i)	1st B1 for Quota advantage 2nd B1 for Quota disadvantage	
(ii)	3rd B1 for Systematic Advantage 4th B1 for Systematic Disadvantage	

Question Number	Scheme	Marks
Q2 (a)	Limits are $20.1 \pm 1.96 \times 0.5$	M1 B1
	(19.1, 21.1)	A1cso (3)
(b)	98 % confidence limits are	
	$20.1 \pm 2.3263 \times \frac{0.5}{\sqrt{10}}$	M1 B1
	(19.7, 20.5)	A1A1 (4)
(c)	The growers claim is not correct Since 19.5 does not lie in the interval (19.7, 20.5)	B1 dB1 (2) **[9]**
(a)	M1 for $20.1 \pm z \times 0.5$. Need 20.1 and 0.5 in correct places with no $\sqrt{10}$ B1 for $z = 1.96$ (or better) A1 for awrt 19.1 <u>and</u> awrt 21.1 **but must have scored both M1 and B1** [Correct answer only scores 3/3]	
(b)	M1 for $20.1 \pm z \times \frac{0.5}{\sqrt{10}}$, need to see 20.1, 0.5 and $\sqrt{10}$ in correct places B1 for $z = 2.3263$ (or better) 1st A1 for awrt 19.7 2nd A1 for awrt 20.5 [Correct answer only scores M1B0A1A1]	
(c)	1st B1 for rejection of the claim. Accept "unlikely" or "not correct" 2nd dB1 Dependent on scoring 1st B1 in this part for rejecting grower's claim for an argument that supports this. Allow comment on <u>their</u> 98% CI from (b)	

Question Number	Scheme	Marks
Q3 (a)		

	A	B	C	D	E	F	G	H	I	J
BMI	1	6	3	8	4	5	7	2	9	10
or	10	5	8	3	7	6	4	9	2	1
Finishing position	3	5	1	9	6	4	10	2	7	8
d^2	4	1	4	1	4	1	9	0	4	4

Question Number	Scheme	Marks
	(table above)	M1
	$\sum d^2 = 32 \quad (298)$	M1
	$r_s = 1 - \dfrac{6\times 32}{10\times 99}$	M1 A1ft
	$= 0.80606\ldots \quad (-0.80606) \qquad$ accept $\pm\dfrac{133}{165}$ **<u>awrt $\pm$ 0.806</u>**	A1 (5)
(b)	$H_0 : \rho = 0,\ H_1 : \rho > 0,$	B1 B1
	Critical value is $(\pm)0.5636$	B1
	$(0.806 > 0.5636$ therefore) in critical region/ reject H_0	M1
	The lower the BMI the higher the position in the race./ support for doctors belief	A1ft (5)
(c)	The position is already ranked OR Position is not Normally distributed	B1 (1) **[11]**
(a)	1st M1 for attempt to rank BMI scores	
	2nd M1 for attempt at $\sum d^2$ (<u>must</u> be using ranks)	No ranking can score 3rd M1 only
	3rd M1 for use of the correct formula with their $\sum d^2$. If answer is not correct an expression is required.	
	1st A1ft for a correct expression. ft their $\sum d^2$ but only if all 3 Ms are scored	
	2nd A1 awrt $\pm$ 0.806 (but sign must be compatible with their $\sum d^2$)	
(b)	2nd B1 for $\rho > 0$ (or <0 but must be one tail and consistent with their ranking)	No H_1 assume one-tail for 3rd B1
	3rd B1 for critical value that is compatible with their H_1. If one-tail must be $\pm$ 0.5636 if two-tail must be $\pm$ 0.6485 [Condone wrong sign]	
	M1 for a correct statement relating their r_s with their cv. e.g. "reject H_0", "in critical region", "significant result" May be implied by a correct comment	
	A1ft for correct comment in context. Must mention low/high BMI and race/fitness <u>or</u> doctor's belief. Comment should be <u>one</u>-tailed. Allow positive <u>correlation</u> between… but <u>NOT</u> …positive <u>relationship</u>…	
(c)	B1 for a correct and relevant comment either based on the fact that the data was originally partially ordered <u>or</u> on the underlying normal assumption "Quicker" or "easier" score B0	

Question Number	Scheme	Marks
Q4	$X \sim \text{N}(55, 3^2)$ therefore $\bar{X} \sim \text{N}(55, \frac{9}{8})$	B1 B1
	$\text{P}(\bar{X} > 57) = \text{P}\left(Z > \frac{57-55}{\sqrt{\frac{9}{8}}}\right) \quad = \text{P}(Z > 1.8856\ldots)$	M1
	$= 1 - 0.9706$	M1
	$= 0.0294$ **<u>0.0294~0.0297</u>**	A1
		[5]
	1^{st} B1 for $\bar{X} \sim$ normal and $\mu = 55$, may be implied but must be $\bar{X}$ 2^{nd} B1 for $\text{Var}(\bar{X})$ or st. dev of $\bar{X}$ e.g. $\bar{X} \sim \text{N}(55, \frac{9}{8})$ or $\bar{X} \sim \text{N}\left(55, \left(\frac{3}{\sqrt{8}}\right)^2\right)$ for B1B1 Condone use of X if they clearly mean $\bar{X}$ so $X \sim \text{N}\left(55, \frac{9}{8}\right)$ is OK for B1B1 1^{st} M1 for an attempt to standardize with 57 and mean of 55 and their st. dev. $\neq 3$ 2^{nd} M1 for 1 - tables value. Must be trying to find a probability < 0.5 A1 for answers in the range 0.0294~0.0297	
ALT	$\sum_{1}^{8} X_i \sim \text{N}\left(8 \times 55, 8 \times 3^2\right)$ 1^{st} B1 for $\sum X \sim$ normal and mean $= 8 \times 55$ 2^{nd} B1 for variance $= 8 \times 3^2$ 1^{st} M1 for attempt to standardise with 57×8, mean of 55×8 and their st dev $\neq 3$	

Question Number	Scheme	Marks

Q5 (a)

$$\lambda = \frac{0\times40+1\times33+2\times14+3\times8+4\times5}{100} = 1.05$$

Marks: M1 A1 (2)

(b)

Using Expected frequency = 100 × P($X = x$) = 100 × $\frac{e^{-1.05}1.05^x}{x!}$ gives — M1

$r = 36.743$ — awrt 36.743 or 36.744 — A1

$s = 19.290$ — 19.29 or awrt 19.290 — A1 (3)

(c)

H_0 : Poisson distribution is a suitable model
H_1 : Poisson distribution is not a suitable model — B1

Number of goals	Frequency	Expected frequency	
0	40	34.994	
1	33	36.743	
2	14	19.290	
3	8	6.752	8.972443
≥ 4	5	2.221	

M1

$\upsilon = 4 - 1 - 1 = 2$ — B1ft

CR : $\chi_2^2(0.05) > 5.991$ — B1

$$\sum\frac{(O-E)^2}{E} = \frac{(40-34.9937)^2}{34.9937} + \ldots + \frac{(13-8.972443)^2}{8.972443}$$ — M1

[=0.7161…+0.3813…+1.4508…+1.80789..]

= 4.356. (ans in range 4.2 – 4.4) — A1

Not in critical region
Number of goals scored can follow a Poisson distribution / managers claim is justified — A1 ft (7)

[12]

(a) M1 for an attempt to find the mean- at least 2 terms on numerator seen
Correct answer only will score both marks

(b) M1 for use of correct formula (ft their mean). 1st A1 for r , 2nd A1 for s (19.29 OK)

(c) 1st B1 Must have both hypotheses and mention Poisson at least once
inclusion of their value for mean in hypotheses is B0 but condone in conclusion

1st M1 for an attempt to pool ≥ 4

2nd B1ft for n -1-1 = 2 i.e realising that they must subtract 2 from their n

3rd B1 for 5.991 only

2nd M1 for an attempt at the test statistic, at least 2 correct expressions/values (to 3sf)

1st A1 for answers in the range 4.2~4.4

2nd A1 for correct comment in context based on their test statistic and their cv that mentions goals or manager. Dependent on 2nd M1
Condone mention of Po(1.05) in conclusion
Score A0 for inconsistencies e.g. "significant" followed by "manager's claim is justified"

Question Number	Scheme	Marks
Q6 (a)	$\mu_U \sim$ mean length of upper shore limpets, $\mu_L \sim$ mean length of lower shore limpets $H_0 : \mu_u = \mu_L$ $H_1 : \mu_u < \mu_L$ both	B1
	s.e. $= \sqrt{\dfrac{0.42^2}{120} + \dfrac{0.67^2}{150}}$	M1 A1
	$= 0.0668$	
	$z = \dfrac{5.05 - 4.97}{0.0668} = (\pm)1.1975$ awrt $\pm$ **<u>1.20</u>**	dM1 A1
	Critical region is $z \geq 1.6449$, or probability = awrt (0.115 or 0.116) $z = \pm$ 1.6449	B1
	($1.1975 < 1.6449$) therefore not in critical region / accept H_0/not significant (or $P(Z \geq 1.1975) = 0.1151$, $0.1151 > 0.05$ or z not in critical region)	M1
	There is no evidence that the limpets on the upper shore are shorter than the limpets on the lower shore.	A1 (8)
(b)	Assume the populations or variables are independent Standard deviation of sample = standard deviation of population [Mention of <u>Central Limit Theorem</u> does <u>NOT</u> score the mark]	B1 B1 (2) **[10]**
(a)	1st B1 If μ_1, μ_2 used then it must be clear which refers to upper shore. Accept sensible choice of letters such as u and l. 1st M1 Condone minor slips e.g. $\dfrac{0.67^2}{120}$ or $\dfrac{0.67}{150} + \dfrac{0.42^2}{120}$ etc i.e. swapped n or one sd and one variance but M0 for $\sqrt{\dfrac{0.67}{150} + \dfrac{0.42}{120}}$ 1st A1 can be scored for a fully correct expression. May be implied by awrt 1.20 2nd dM1 is dependent upon the 1st M1 but can ft their se value if this mark is scored. 2nd A1 for awrt ($\pm$) 1.20 3rd M1 for a correct statement based on their z value and their cv. No cv is M0A0 If using probability they must compare their p (<0.5) with 0.05 (o.e) so can allow 0.884< 0.95 to score this 3rd M1 mark. May be implied by their contextual statement and M1A0 is possible. 3rd A1 for a correct comment to accept null hypothesis that mentions <u>length</u> of <u>limpets</u> on the two <u>shores</u>.	
(b)	1st B1 for one correct statement. Accept ”samples are independent” 2nd B1 for both statements	

Question Number	Scheme	Marks
Q7 (a)	Estimate of Mean $= \frac{600.9}{5} = 120.18$	M1A1
	Estimate of Variance $= \frac{1}{4}\{72216.31 - \frac{600.9^2}{5}\}$ or $\frac{0.148}{4} = 0.037$	M1 A1ft A1 (5)
(b)	$P(-0.05 < \mu - \hat{\mu} < 0.05) = 0.90$ or $P(-0.05 < \overline{X} - \mu < 0.05) = 0.90$ [$\leq$ is OK]	B1
	$\frac{0.05}{\frac{0.2}{\sqrt{n}}} = 1.6449$	M1 A1
	$n = \frac{1.6449^2 \times 0.2^2}{0.05^2}$	dM1
	$n = 43.29\ldots$	A1
	$n = 44$	A1 (6) **[11]**
(a)	1st M1 for an attempt at $\sum x$ (accept 600 to 1sf) 1st A1 for $\frac{600.9}{5}$ = awrt 120 or awrt 120.2. No working give M1A1 for awrt 120.2 2nd M1 for the use of a correct formula including a reasonable attempt at $\sum x^2$ (Accept 70 000 to 1sf) or $\sum\left(x-\bar{x}\right)^2 = 0.15$ (to 2 dp) 2nd A1ft for a correct expression with correct $\sum x^2$ but can ft their <u>mean</u> (for expression - no need to check values if it is incorrect) 3rd A1 for 0.037 Correct answer with no working scores 3/3 for variance	
(b)	B1 for a correct probability statement <u>or</u> "width of 90% CI = $0.05 \times 2 = 0.1$" 1st M1 for $\frac{0.05}{\frac{0.2}{\sqrt{n}}} = z$ value <u>or</u> $2 \times \frac{0.2}{\sqrt{n}} \times z = 0.1$ Condone 0.5 instead of 0.05 <u>or</u> missing 2 <u>or</u> 0.05 for 0.1 for M1 1st A1 for a correct equation including 1.6449 2nd dM1 Dependent upon 1st M1 for rearranging to get $n = \ldots$ Must see "squaring" 2nd A1 for n = awrt 43.3 3rd A1 for rounding up to get $n = 44$ Using e.g.1.645 instead of 1.6449 can score all the marks except the 1st A1	1st B1 may be implied by 1st A1 scored or correct equation.

Question Number	Scheme		Marks
Q8 (a)	$E(4X - 3Y) = 4E(X) - 3E(Y)$		M1
	$= 4\times 30 - 3\times 20$		
	$= 60$		A1 (2)
(b)	$\text{Var}(4X - 3Y) = 16\text{ Var }(X) + 9\text{ Var }(Y)$	16 or 9; adding	M1; M1
	$= 16\times 9 + 9\times 4$		
	$= 180$		A1 (3)
(c)	$E(B) = 80$		B1
	$\text{Var }(B) = 16$		B1
	$E(B - A) = 20$	$E(B)$-$E(A)$	M1
	$\text{Var }(B - A) = 196$	ft on 180 and 16	A1ft
	$P(B - A > 0) = P\left(Z > \frac{-20}{\sqrt{196}}\right) = [P(Z > -1.428...)]$	stand. using their mean and var	dM1
	$= 0.923\ \ldots$	awrt 0.923 – 0.924	A1 (6) **[11]**
(a)	M1 for correct use of $E(aX + bY)$ formula		
(b)	1st M1 for $16\text{Var}(X)$ <u>or</u> $9\text{Var}(Y)$ 2nd M1 for <u>adding</u> variances Key points are the 16, 9 and +. Allow slip e.g using $\text{Var}(X)=4$ etc to score Ms		
(c)	1st M1 for attempting $B - A$ and $E(B - A)$ or $A - B$ and $E(A - B)$ This mark may be implied by an attempt at a correct probability e.g. $P\left(Z > \frac{0-(80-60)}{\sqrt{180+16}}\right)$. To be implied we must see the "0" 1st A1ft for $\text{Var}(B - A)$ can ft their $\text{Var}(A) = 180$ and their $\text{Var}(B) = 16$ 2nd dM1 Dependent upon the 1st M1 in part (c). for attempting a correct probability i.e. $P(B$-$A>0)$ or $P(A$-$B < 0)$ and standardising with their mean and variance. They must standardise properly with the 0 to score this mark 2nd A1 for awrt 0.923 ~ 0.924		

June 2009
6686 Statistics S4
Mark Scheme

Question Number	Scheme		Marks
Q1	H_0: $\mu = 5$; H_1: $\mu < 5$	both	B1
	CR: $t_9(0.01) > 2.821$		B1
	$\bar{x} = 4.91$		B1
	$s^2 = \frac{1}{9}\left(241.2 - \frac{49.1^2}{10}\right) = 0.0132222$	s= awrt 0.115	M1 A1
	$t = \frac{\lvert 4.91 - 5\rvert}{\frac{\sqrt{0.013222}}{\sqrt{10}}} = \pm 2.475$	2.47 – 2.48	M1 A1
	Since 2.475 is not in the critical region there is insufficient evidence to reject H_0 and conclude that the mean diameter of the bolts is not less than (not equal to) 5mm.		A1ft
			[8]

Question Number	Scheme	Marks
Q2 (a)	The differences are normally distributed	B1 (1)
(b)	The data is collected in pairs or small sample size and variance unknown or samples not independent	B1 (1)
(c)	d: 2.5, 1.6, 1.6, –1.9, –0.6, 4.5 at least 2 correct	M1
	$(\Sigma d = 7.7, \Sigma d^2 = 35.59)$ $\bar{d} = \pm 1.2833$, sd = 2.2675. (Var = 5.141)	A1, A1
	H_0: $\mu_d = 0$, H_1: $\mu_d > 0$ (H_1: $\mu_d < 0$ if d - 2.5, -1.6, -1.6 etc) both depend on their d's	B1
	$t = \dfrac{\pm 1.2833\sqrt{6}}{2.2675} = \pm 1.386......$ formula and substitution, 1.38 – 1.39	M1, A1
	Critical value $t_5(5\%) = 2.015$ (1 tail)	B1
	Not significant. Insufficient evidence to support that the device reduces CO_2 emissions.	A1 ft (8)
(d)	The idea that the device reduces $C0_2$ emissions has been rejected when in fact it does reduce emissions. OR Concluding that the device does not reduce emissions when in fact it does (if not in context can get B1 only)	B1 B1 (2)
		[12]
	(b) Allow because the same car has been used (c) awrt ± 1.28 , 2.27	

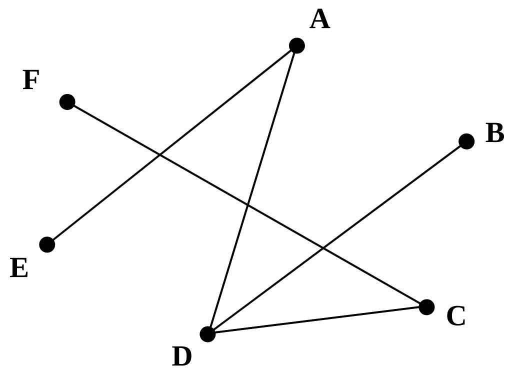
Question Number	Scheme	Marks
3 (a)	Size is the probability of H_0 being rejected when it is in fact true. or P(reject H_0/ H_0 is true) oe	B1 (1)
(b)	The power of the test is the probability of rejecting H_0 when H_1 is true. or P(rejecting H_0/H_1 is true) / P(rejecting H_0/H_0 is false) oe	B1 (1)
(c)	X ~B(12,0.5)	B1
	$P(X \leq 2) = 0.0193$	M1
	$P(X \geq 10) = 0.0193$	
	$\therefore$ critical region is $\{X \leq 2 \cup X \geq 10\}$	A1A1 (4)
(d)(i)	P(Type II error) $= P(3 \leq X \leq 9 \mid p = 0.4)$	M1
	$= P(X \leq 9) - P(X \leq 2)$	M1dep
	$= 0.9972 - 0.0834$	
	$= 0.9138$	A1
(ii)	Power $= 1 - 0.9138$ $= 0.0862$	B1 ft (4)
(e)	Increase the sample size	B1
	Increase the significance level/larger critical region	B1 (2)
		[12]
Notes	(d) (i) first M1 for either correct area or follow through from their critical region 2nd M1 dependent on them having the first M1. for finding their area correctly A1 cao (ii) B1 follow through from their (i)	

Question Number	Scheme	Marks
Q4 (a)	$\text{H}_0 : \sigma_A^2 = \sigma_B^2,\ \text{H}_1 : \sigma_A^2 \neq \sigma_B^2$	B1
	critical values $F_{12,8} = 3.28$ and $\frac{1}{F_{8,12}} = 0.35$	B1
	$\frac{s_B^2}{s_A^2} = 2.40 \quad \left(\frac{s_A^2}{s_B^2} = 0.416\right)$	M1A1
	Since 2.40 (0.416) is not in the critical region we accept H_0 and conclude there is no evidence that the two variances are different.	A1ft (5)
(b)	$S_p^2 = \frac{8 \times 1.02 + 12 \times 2.45}{20}$	M1
	$= 1.878$	A1
	$(27.94 - 25.54) \pm 2.086 \times \sqrt{1.878} \times \sqrt{\frac{1}{9} + \frac{1}{13}}$	B1M1 A1ft
	$(1.16,\ 3.64)$	A1 A1 (7)
(c)	To calculate the confidence interval the variances need to be equal.	B1
	In part (a) the test showed they are equal.	B1 (2)
		[14]

Question Number	Scheme	Marks
Q5 (a)	95% confidence interval for μ is 2.145	B1
	$560 \pm t_{14}(2.5\%)\sqrt{\frac{25.2}{15}} = 560 \pm 2.145\sqrt{\frac{25.2}{15}} = (557.2, 562.8)$	M1 A1 A1 (4)
(b)	95% confidence interval for σ^2 is	
	$5.629 < \frac{14 \times 25.2}{\sigma^2} < 26.119$	B1, M1, B1
	$\sigma^2 < 62.675$ $\sigma^2 > 13.507$	
	$13.507 < \sigma^2 < 62.675$ awrt 13.5, 62.7	A1, A1 (5)
(c)	Require $\mathrm{P}(X > 565) = \mathrm{P}\left(Z > \frac{565-\mu}{\sigma}\right)$ to be as large as possible OR	
	$\frac{565-\mu}{\sigma}$ to be as small as possible; both imply highest σ and μ.	M1
	$\frac{565-562.8}{\sqrt{62.675}} = 0.28$	M1A1
	$\mathrm{P}(Z > 0.28) = 1 - 0.6103 = 0.3897$ awrt 0.39 – 0.40	M1 A1 (5)
	(c) M1 for using their largest σ and μ M1 for using $\frac{x-\mu}{\sigma}$ M1 1 – their prob	[14]

Question Number	Scheme	Marks
Q6 (a)	$\text{E}(\tfrac{2}{3}X_1+\tfrac{1}{2}X_2+\tfrac{5}{6}X_3)=\tfrac{2}{3}\times\tfrac{k}{2}+\tfrac{1}{2}\times\tfrac{k}{2}+\tfrac{5}{6}\times\tfrac{k}{2}=k$	M1 A1
	$\text{E}(X_1+X_2+X_3)=k \Rightarrow$ unbiased	B1
		(3)
(b)	$\text{E}(aX_1+bX_2)=a\dfrac{k}{2}+b\dfrac{k}{2}=k$	M1
	$a+b=2$	A1
	$\text{Var}(aX_1+bX_2)=a^2\dfrac{k^2}{12}+b^2\dfrac{k^2}{12}$	M1A1
	$=a^2\dfrac{k^2}{12}+(2-a)^2\dfrac{k^2}{12}$	M1
	$=(2a^2-4a+4)\dfrac{k^2}{12}$	
	$=(a^2-2a+2)\dfrac{k^2}{6}$ (*) since answer given	A1 cso
(c)		(6)
	Min value when $(2a-2)\dfrac{k^2}{6}=0$ $\quad \dfrac{\text{d}}{\text{d}a}(\text{Var})=0$, all correct, condone missing $\dfrac{k^2}{6}$	M1A1
	$\Rightarrow 2a-2=0$ $a=1, b=1.$	A1A1
	$\dfrac{\text{d}^2(\text{Var})}{\text{d}a^2}=\dfrac{2k^2}{6}>0$ since $k^2>0$ therefore it is a minimum	M1
	min variance $=(1-2+2)\dfrac{k^2}{6}$ $=\dfrac{k^2}{6}$	B1
		(6)
	Alternative	
	$\dfrac{k^2}{6}(a-1)^2-\dfrac{k^2}{6}+\dfrac{2k^2}{6}$	M1 A1
	$\dfrac{k^2}{6}(a-1)^2+\dfrac{k^2}{6}$	M1
	Min when $\dfrac{k^2}{6}(a-1)^2=0$	A1A1
	$a=1$ $b=1$	B1
	min var $=k^2/6$	

June 2009
6689 Decision Mathematics D1
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	AD, AE, DB; DC, CF	M1 A1; A1 (3)
(b)		B1 (1)
(c)	Weight 595 (km)	B1 (1) **[5]**
	Notes: (a) 1M1: Using Prim – first 2 arcs probably but condone starting from another vertex. 1A1: first three arcs correct 2A1: all correct. (b) 1B1: CAO (c) 1B1: CAO condone lack of km. <u>Apply the misread rule, if not listing arcs or not starting at A.</u> So for M1 (only) Accept numbers across the top (condoning absence of 6) Accept full vertex listing Accept full arc listing starting from vertex other than A [AD AE DB DC CF] {1 4 5 2 3 6} ADEBCF BD AD AE CD CF {3 1 5 2 4 6} BDAECF CD AD AE BD CF {3 5 1 2 4 6} CDAEBF DA AE DB CD CF {2 4 5 1 3 6} DAEBCF EA AD DB DC CF {2 4 5 3 1 6} EADBCF FC CD AD AE BD {4 6 2 3 5 1} FCDAEB	

Question Number	Scheme	Marks
Q2		
(a)	$\frac{230}{60} = 3.8\dot{3}$ so 4 needed	M1 A1 (2)
(b)	Bin 1: 32 17 9 Bin 2: 45 12 Bin 3: 23 28 Bin 4: 38 16 Bin 5: 10	M1 A1 A1 A1 (4)
(c)	e.g. Bin 1: 32 28 Bin 2: 38 12 10 Bin 3: 45 9 Bin 4: 23 17 16	M1 A1 A1 (3)
		[9]
	Notes: (a) 1M1: Their 230 divided by 60, some evidence of correct method 3.8 enough. 1A1: cso 4. (b) 1M1: Use of first fit. Probably 32, 45 and 17 correctly placed. 1A1: 32, 45, 17, 23, 38 and 28 placed correctly 2A1: 32, 45, 17, 23, 38, 28, 16, 9 placed correctly. 3A1: cao (c) 1M1: Use of full bin – at least one full bin found and 5 numbers placed. 1A1: 2 full bins found Eg [32+28 and 38+12+10] [23+28+9 and 16+12+32] [32+28 and 23+16+12+9] [38+12+10 and 23+28+9] 2A1: A 4 bin solution found. **Special case for (b) misread using first fit decreasing.** Give M1A1 (max) Bin 1: 45 12 Bin 2: 38 17 Bin 3: 32 28 Bin 4: 23 16 10 9 M1 for placing 45, 38, 32, 28 and 23 correctly A1 for cao.	

Question Number	Scheme	Marks	
Q3 (a)	H – 2 = M – 5 = R – 4 change status to give	M1 A1	
	C = 3 (E unmatched) H = 2 M = 5 R = 4 S = 1	A1	(3)
(b)	e.g. C is the only person who can do 3 and the only person who can do 6	B1	(1)
(c)	e.g. E – 5 = M – 2 = H – 1 = S – 3 = C – 6 change status to give	M1 A1	
	C = 6 E = 5 H = 1 M = 2 R = 4 S = 3	A1	(3)
			[7]
	Notes: (a) 1M1: Path from H to 4 1A1: correct path and change status 2A1: CAO must follow from correct path. (b) 1B1: CAO or e.g reference to E 5 M 2 H 1 S (c) 1M1: Path from E to 6 1A1: CAO do not penalise lack of change status a second time. 2A1: CAO must follow from a correct path		

Question Number	Scheme	Marks

Q4

M	J	E	K	H	**B**	L	P	N	D	**B**
B	M	J	E	K	**H**	L	P	N	D	**H**
B	E	**D**	***H***	M	J	K	**L**	P	N	**D L**
B	***D***	E	***H***	J	**K**	***L***	M	**P**	N	**(E) K P**
B	***D***	***E***	***H***	**J**	***K***	***L***	M	**N**	***P***	**(J) N**
B	***D***	***E***	***H***	***J***	***K***	***L***	**M**	***N***	***P***	**(M)**

Marks: M1 1A1; 2A1ft; 3A1ft

Sort completed — 4A1 (5)

$\left[\frac{1+10}{2}\right]=6$ Katie reject left — M1

$\left[\frac{7+10}{2}\right]=9$ Natsuko reject right — 1A1

$\left[\frac{7+8}{2}\right]=8$ Miri reject right — 2A1ft

7 = Louis name found — 3A1 (4)

[9]

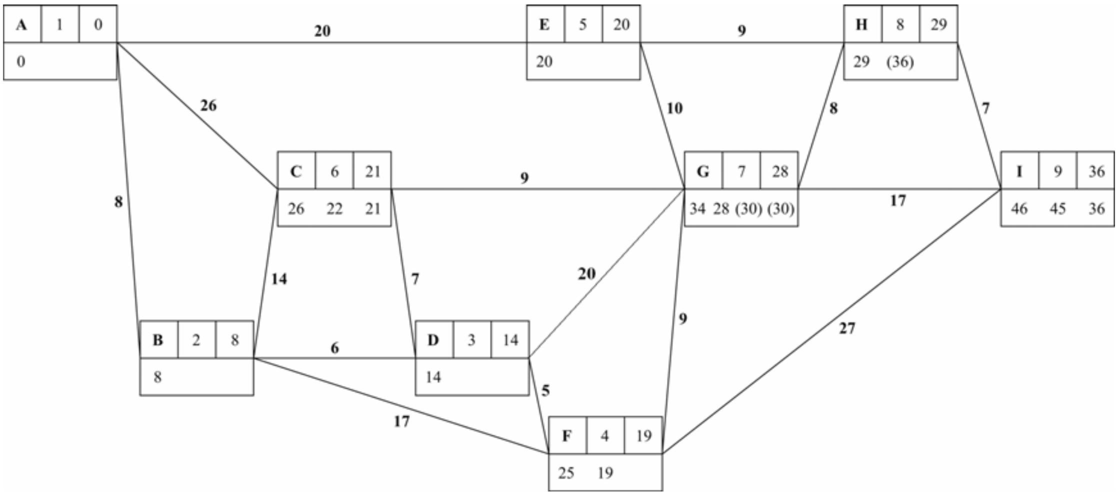
Notes:

(a) 1M1: quick sort, pivots, p, identified, two sublists one <p one >p.
If choosing one pivot only per iteration, M1 only.
1A1: first pass correct, next pivot(s) chosen consistently.
2A1ft: second pass correct, next pivot(s) chosen consistently
3A1ft: third pass correct, next pivot(s) chosen consistently
4A1: cso List re-written or end statement made or each element been chosen as a pivot.

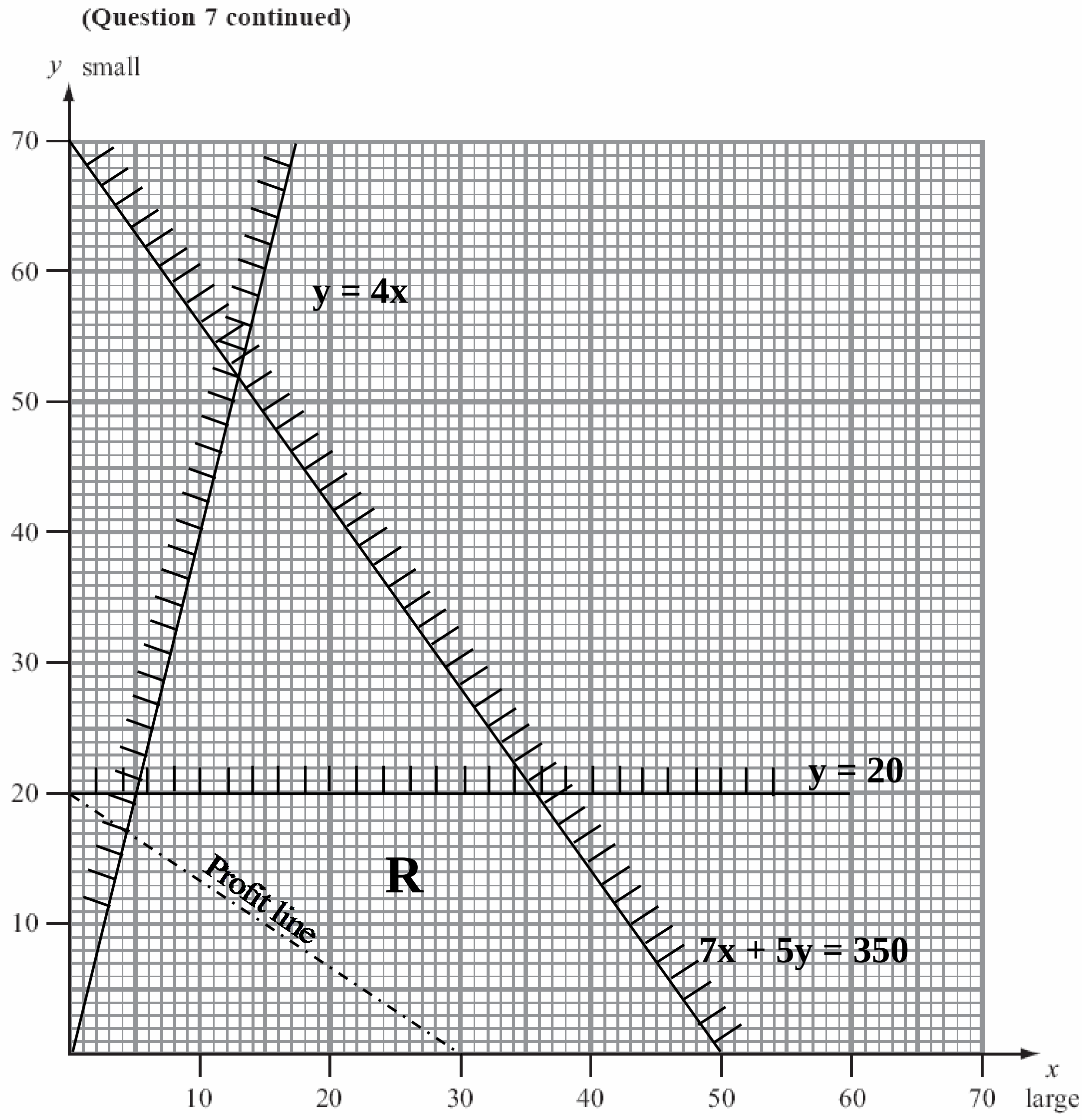
(b) 1M1: binary search, choosing pivot rejecting half list.
If using unordered list then M0.
If choosing J M1 ony
1A1: first two passes correct, condone 'sticky'pivots here, bod.
2A1ft: third pass correct, pivots rejected.
3A1: cso, including success statement.

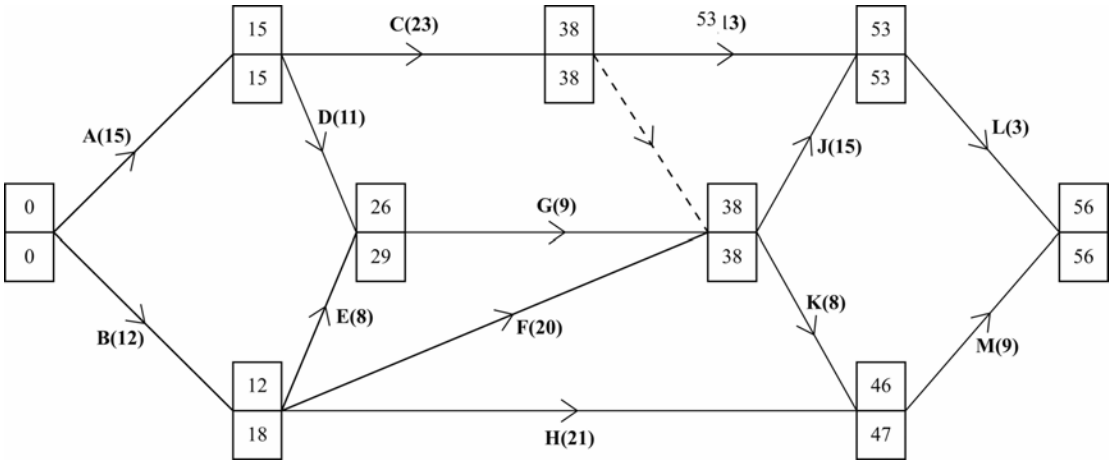
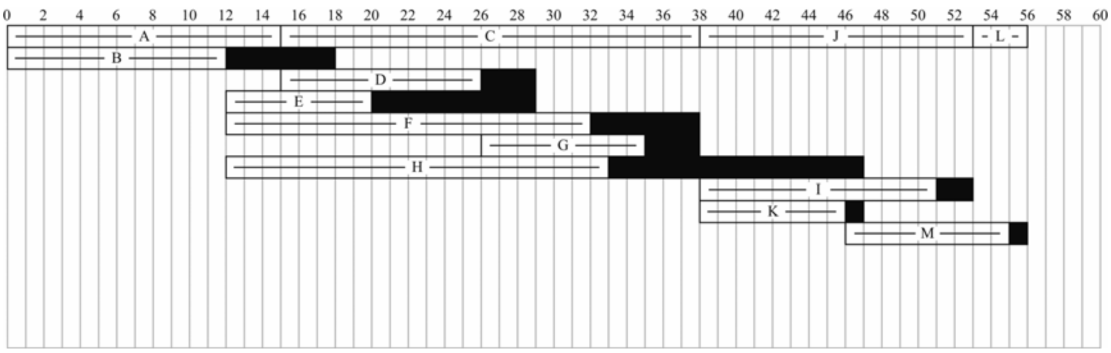
Special case for (b) – If just one letter out of order, award maximum of M1A1A0A0

Question Number	Scheme	Marks
Q5 (a)	CD + EG = 45 + 38 = 83 CE + DG = 39 + 43 = 82 $\leftarrow$ CG + DE = 65 + 35 = 100 Repeat CE and DG Length 625 + 82 = 707 (m)	M1 1A1 2A1 3A1 4A1ft 5A1ft (6)
	DE (or 35) is the **smallest** So finish at C. New route 625 + 35 = 660 (m)	M1 A1ft A1ft=1B1 (3) **[9]**
	Notes: (a) 1M1: Three pairings of their four odd nodes 1A1: one row correct 2A1: two rows correct 3A1: three rows correct 4A1ft: ft their least, but must be the correct shortest route arcs on network. (condone DG) 5A1ft: 625 + their least = a number. Condone lack of m (b) 1M1: Identifies their shortest from a choice of at least 2 rows. 1A1ft: ft from their least or indicates C. 2A1ft = 1Bft: correct for their least. (Indept of M mark)	

Question Number	Scheme	Marks
Q6 (a)		M1 1A1 2A1ft 3A1ft 4A1ft
	Route: A E H I	5A1
(b)	Shortest distance from A to G is 28 km	B1ft
		[7]
	Notes: (a) 1M1: Small replacing big in the working values at C or F or G or I 1A1: Everything correct in boxes at A, B, D and F 2A1ft: ft boxes at E and C handled correctly but penalise order of labelling only once 3A1ft: ft boxes at G and H handled correctly but penalise order of labelling only once 4A1ft: ft boxes at I handled correctly but penalise order of labelling only once 5A1: route cao A E H I (b) 1B1ft: ft their final label at G condone lack of km	

Question Number	Scheme	Marks
Q7 (a)	$7x+5y \le 350$	M1 A1 (2)
(b)	$y \le 20$ e.g. make at most 20 small baskets $y \le 4x$ e.g. the number of small (y) baskets is at most 4 times the number of large baskets (x). {E.g if y = 40, x =10, 11, 12 etc. or if x = 10, y = 40, 39, 38}	B1 B1 (2)
(c)	(see graph next page) Draw three lines correctly Label R	B3,2,1,0 B1 (4)
(d)	(P=) $2x + 3y$	B1 (1)
(e)	Profit line or point testing. $x = 35.7$ $y = 20$ precise point found. Need integers so optimal point in R is (35, 20); Profit (£)130	M1 A1 B1 B1;B1 (5)
		[14]
	Notes: (a) 1M1: Coefficients correct (condone swapped x and y coefficients) need 350 and any inequality 1A1: cso. (b) 1B1: cao 2B1: cao, test their statement, need both = and < aspects. (c) 1B1: One line drawn correctly 2B1: Two lines drawn correctly 3B1: Three lines drawn correctly. Check (10, 40) (0, 0) and axes 4B1: R correct, but allow if one line is slightly out (1 small square). (d) 1B1: cao accept an expression. (e) 1M1: Attempt at profit line or attempt to test at least two vertices in their feasible region. 1A1: Correct profit line or correct testing of at least three vertices. **Point testing:** (0,0) P= 0; (5,20) P = 70; (50,0) P = 100 $\left(35\frac{5}{7},20\right)=\left(\frac{250}{7},20\right)$ P = $131\frac{3}{7}=\frac{920}{7}$ also (35, 20) P = 130. Accept (36,20) P = 132 for M but not A. **Objective line:** Accept gradient of 1/m for M mark or line close to correct gradient. 1B1: cao – accept x co-ordinates which round to 35.7 2B1: cao 3B1: cao	

Question Number	Scheme	Marks
(c)		

Question Number	Scheme	Marks
Q8 (a)		M1 A1 M1 A1 (4)
(b)	A C J L	B1 (1)
(c)	Total float for M = 56(ft) – 46 – 9 = 1 Total float for H = 47 – 12 – 21 = 14	M1 A1ft B1 (3)
(d)		M1 A1 M1,A1 (4)
(e)	1pm day 16: C 1pm day 31: C F G H	B1ft B2ft,1ft,0 (3) **[15]**

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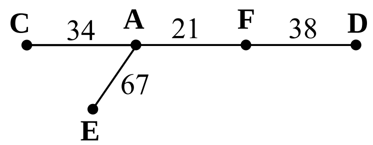
6690 Decision Mathematics D2
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	There are more tasks than people.	B1 (1)
(b)	Adds a row of zeros	B1 (1)
(c)	$\begin{bmatrix} 15 & 11 & 14 & 12 \\ 13 & 8 & 17 & 13 \\ 14 & 9 & 13 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 0 & 3 & 1 \\ 5 & 0 & 9 & 5 \\ 5 & 0 & 4 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} ; \rightarrow \begin{bmatrix} 3 & 0 & 2 & 0 \\ 4 & 0 & 8 & 4 \\ 4 & 0 & 3 & 5 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	B1;M1A1
	Either $\begin{bmatrix} 3 & 3 & 2 & 0 \\ 1 & 0 & 5 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 4 & 0 & 0 \end{bmatrix}$	M1 A1
	Or $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 6 & 4 \\ 2 & 0 & 1 & 5 \\ 0 & 3 & 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 5 & 3 \\ 1 & 0 & 0 & 4 \\ 0 & 4 & 0 & 2 \end{bmatrix}$	
	J – 4, M – 2, R – 3, (D – 1)	A1 (6)
(d)	Minimum cost is (£)33.	B1 (1)
		[9]

Question Number	Scheme	Marks
Q2 (a)	In the classical problem each vertex must be visited **only** once. In the practical problem each vertex must be visited **at least** once.	B2, 1, 0 (2)
(b)	A F D B E C A {1 4 6 3 5 2 } 21 + 38 + 58 + 36 + 70 + 34 = 257	M1 A1 A1 (3)
(c)	257 is the better upper bound, it is lower.	B1ft (1)
(d)	R.M.S.T.  Lower bound is 160 + 36 + 58 = 254	M1 A1 M1A1 (4)
(e)	Better lower bound is 254, it is higher	B1ft
(f)	$254 < \text{optimal} \le 257$	B1 (2)
		[12]

Notes:

(a) 1B1: Generous, on the right lines bod gets B1
2B1: cao, clear answer.

(b) 1M1:Nearest Neighbour each vertex visited once (condone lack of return to start)
1A1: Correct route cao – must return to start.
2A1: 257 cao

(c) 1B1ft: ft their lowest.

(d) 1M1: Finding correct RMST (maybe implicit) 160 sufficient
1A1: cao tree or 160.
2M1:Adding 2 least arcs to B, 36 and 58 only
2A1: 254

(e) 1B1ft: ft their highest

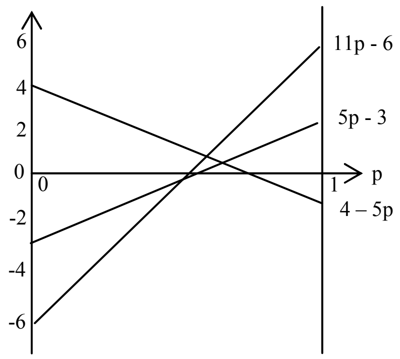
(f) 1B1: cao

Question Number	Scheme	Marks
Q3 (a)	Row minima {-5, -4, -2} row maximin = -2 Column maxima {1, 6, 13} col minimax = 1 $-2 \neq 1$ therefore not stable.	M1 A1 A1 (3)
(b)	Column 1 dominates column 3, so column 3 can be deleted.	B1 (1)
(c)	(table below)	B1 B1 (2)
(d)	Let B play row 1 with probability p and row 2 with probability (1-p) If A plays 1, B's expected winnings are 11p – 6 If A plays 2, B's expected winnings are 4 – 5p If A plays 3, B's expected winnings are 5p – 3	M1 A1
	(graph below)	M1 A1
	$5p-3=4-5p$ $10p=7$ $p=\frac{7}{10}$	M1
	B should play 1 with a probability of 0.7 2 with a probability of 0.3 and never play 3	A1
	The value of the game is 0.5 to B	A1 (7)
		[13]

Table for (c):

	A plays 1	A plays 2	A plays 3
B plays 1	5	-1	2
B plays 2	-6	4	-3

Graph for (d):



Question Number	Scheme	Marks
Q4 (a)	Value of cut $C_1 = 34$; Value of cut $C_2 = 45$	B1; B1 (2)
(b)	S B F G T or S B F E T – value 2 Maximum flow = 28	M1 A1 A1=B1 (3) **[5]**
	Notes: (a) 1B1: cao 2B1: cao (b) 1M1: feasible flow-augmenting route and a value stated 1A1: a correct flow-augmenting route and value 1A1= B1: cao	
Q5 (a)	$x = 0,\ y = 0,\ z = 2$	B2,1,0 (2)
(b)	$P - 2x - 4y + \frac{5}{4}r = 10$	M1 A1 (2) **[4]**
	Notes: (a) 1B1: Any 2 out of 3 values correct 2B1: All 3 values correct. (b) 1M1: One equal sign, modulus of coefficients correct. All the right ingredients. 1A1: cao – condone terms of zero coefficient	

Question Number	Scheme	Marks
Q6 (a)	The supply is equal to the demand	B1 (1)
(b)	(see table below)	B1 (1)

	A	B	C
X	16	6	
Y		9	8
Z			15

(c)

	A	B	C
X	16- θ	6+ θ	
Y		9- θ	8+ θ
Z	θ		15- θ

M1 A1

Value of θ = 9, exiting cell is YB — A1 (3)

(d)

		17	8	20
		A	B	C
0	X	7	15	
-5	Y			17
-11	Z	9		6

M1 A1

XC = 7 – 0 – 20 = -13
YA = 16 + 5 – 17 = 4
YB = 12 + 5 – 8 = 9
ZB = 10 + 11 – 8 = 13

A1 (3)

	A	B	C
X	7- θ	15	θ
Y			17
Z	9+ θ		6- θ

Value of θ = 6, entering cell XC, exiting cell ZC — M1 A1

	A	B	C
X	1	15	6
Y			17
Z	15		

A1 (3)

Cost (£) 524 — B1 (1)

[12]

Question Number	Scheme	Marks

Q7

(a)

Stage	State (in £1000s)	Action (in £1000s)	Dest. (in £1000s)	Value (in £1000s)	Marks
	250	250	0	300*	
1	200	200	0	240*	
	150	150	0	180*	
	100	100	0	120*	
	50	50	0	60*	
	0	0	0	0*	
	250	280	0	200 + 0 = 280	
		200	50	235 + 60 = 295	
		150	100	190 + 120 = 310*	
		100	150	125 + 180 = 305	1M1 A1
		50	200	65 + 240 = 305	
		0	250	0 + 300 = 300	
2	200	200	0	235 + 0 = 235	
		150	50	190 + 60 = 250*	A1
		100	100	125 + 120 = 245	
		50	150	65 + 180 = 245	
		0	200	0 + 240 = 240	
	150	150	0	190 + 0 = 190*	2M1
		100	50	125 + 60 = 185	
		50	100	65 + 120 = 185	A1
		0	150	0 + 180 = 180	
	100	100	0	125 + 0 = 125*	A1
		50	50	65 + 60 = 125*	
		0	100	0 + 120 = 120	
	50	50	0	65 + 0 = 65*	
		0	50	0 + 60 = 60	
	0	0	0	0 + 0 = 0*	3M1
3	250	250	0	300 + 0 = 300	A1ft
		200	50	230 + 65 = 295	
		150	100	170 + 125 = 295	
		100	150	110 + 190 = 300	
		50	200	55 + 250 = 305	
		0	250	0 + 310 = 310*	

Maximum income £310 000 — B1

Scheme	1	2	3
Invest (in £1000s)	100	150	0

B1 (10)

(b)

Stage: Scheme being considered — B1

State: Money available to invest — B1

Action: Amount chosen to invest — B1 (3)

[13]

Question Number	Scheme	Marks
Q8	E.g. Add 6 to make all elements positive $\begin{bmatrix} 4 & 14 & 5 \\ 13 & 10 & 3 \\ 7 & 1 & 10 \end{bmatrix}$	B1
	Let Laura play 1, 2 and 3 with probabilities p_1, p_2 and p_3 respectively Let V = value of game + 6	B1
	e.g. Maximise P = V Subject to:	B1
	$V - 4p_1 - 13p_2 - 7p_3 \le 0$ $V - 14p_1 - 10p_2 - p_3 \le 0$ $V - 5p_1 - 3p_2 - 10p_3 \le 0$ $p_1 + p_2 + p_3 \le 1$ $p_1, p_2, p_3 \ge 0$	M1 A3,2ft,1ft,0 (7)
	Notes: 1B1: Making all elements positive 2B1: Defining variables 3B1: Objective, cao word and function 1M1: At least one constraint in terms of their variables, must be going down columns. Accept = here. 1A1ft: ft their table. One constraint in V correct. 2A1ft: ft their table. Two constraints in V correct. 3A1: CAO all correct .	**[7]**
	Alt using x_i method Now additionally need: let $x_i = \dfrac{p_i}{v}$ for 2B1 minimise $(P) = x_1 + x_2 + x_3 = \dfrac{1}{v}$ subject to: $4x_1 + 13x_2 + 7x_3 \ge 1$ $14x_1 + 10x_2 + x_3 \ge 1$ $5x_1 + 3x_2 + 10x_3 \ge 1$ $x_i \ge 0$	

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